Problem 1 (25 pts) - Submit a report for this problem.

Consider the tank depicted in Figure 1. For a constant inlet flow rate (Q_{in}), the time required to fill the tank to a level H is given by

$$t = \int_0^H \frac{A_T}{Q_{\text{in}} - Q_{\text{out}}} \mathrm{d}h, \tag{1}$$

where A_T is cross-sectional the area of the tank, Q_{in} is the inlet flow rate, and Q_{out} is the outlet flow rate. The outlet flow rate is a function of the height of the liquid in the tank, given by

$$Q_{\text{out}} = A_D \sqrt{2gH}, \tag{2}$$

where A_D is the cross-sectional area of the drain in the bottom of the tank, $g = 9.8\ \frac{\text{m}}{\text{s}^2}$ is the gravitational acceleration, and H is the height of the liquid in the tank. Equation (1) may be integrated analytically to find

$$t = -2\frac{A_T}{A_D\sqrt{2g}}\left(\sqrt{H} + \frac{Q_{\text{in}}}{A_D\sqrt{2g}} \ln\left(\frac{Q_{\text{in}} - A_D\sqrt{2gH}}{Q_{\text{in}}}\right)\right). \tag{3}$$

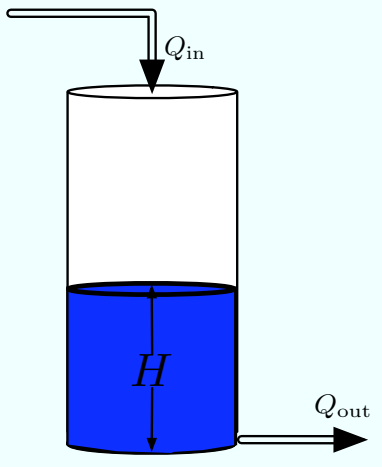

Figure 1: Schematic of a filling tank.

1. (9 pts) Assuming that the tank is 10 meters in diameter and is 10 meters tall, that the outlet is 20 centimeters in diameter, $Q_{\text{in}} = 0.5\ \frac{\text{m}^3}{\text{s}}$, and that the tank is initially empty ($H = 0$ at $t = 0$)

   - (2 pts) Determine how long it will take to fill the tank if $Q_{\text{in}} = 0.5\ \frac{\text{m}^3}{\text{s}}$.
   - (2 pts) Plot the height as a function of time for heights in the range 0 to 10 meters. Note that you can simply calculate $t(H)$ and then plot H versus t.
   - (5 pts) Determine the level of the liquid in the tank after $t = 35$ minutes.

   Solve this part using MATLAB. Submit your MATLAB files and be sure to document them.

2. (9 pts - same distribution as in part 1) Repeat part 1 using Excel. Solve the third part using either goal-seek or solver. In either case, include written instructions in your Excel file on how to solve for the height. Be sure to submit your excel file with your solution.

3. (6 pts) Assuming that the tank and drain have the same dimensions as in part 1, but that the inlet flow rate is decreased to $Q_{\text{in}} = 0.25\,\frac{\text{m}^3}{\text{s}}$, plot H versus t for heights in the range 0 to 10 meters. Do this part in both MATLAB and Excel. **Explain the behavior you observe.** Put some thought into your discussion on this part.

HINTS:

- Be careful with units! I suggest that you keep all units in meters and seconds.
- The natural log of a negative number produces an imaginary number. MATLAB can handle imaginary numbers while Excel cannot. Also, when plotting, MATLAB will ignore the imaginary part of a number.

Submit your brief report along with your MATLAB file that you used to solve the problem.