CHEN 1703 - HOMEWORK 13

Problem 1 (10 pts) - submit a brief report for this problem.

Create a MATLAB script to do the following:

1. (4 pts) Use Matlab's symbolic toolbox to calculate the inverse of a 3×3 matrix, A^{-1}, given $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Use this to calculate $x = A^{-1}b$, where $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.
2. (4 pts) Use `solve` to solve for x in a general 3×3 system $Ax = b$. In other words, write out the three equations implied by the matrix above and solve them using MATLAB's `solve` function.
3. (2 pts) Subtract the result from part 2 from the result from part 1 and show that they are zero.

All of this should be done using a single MATLAB script.

In your report, show the following:

- The analytic solution you obtained from MATLAB for A^{-1}.
- The analytic solution you obtained for x.

Problem 2 (15 pts) - submit a brief report for this problem.

Consider two objects with mass m_1 and m_2 that undergo an elastic collision as depicted in Figure 1.

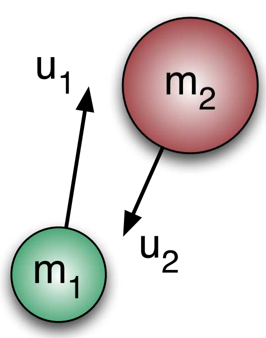

Figure 1: Schematic of a collision between two objects.

Conservation of momentum through the collision relates the initial velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ to the final velocities, $\mathbf{u}_{1f}$ and $\mathbf{u}_{2f}$,

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{u}_{1f} + m_2\mathbf{u}_{2f}, \tag{1}$$

For an elastic collision, kinetic energy is also conserved, which implies

$$m_1\left(\mathbf{u}_1^2 - \mathbf{u}_{1f}^2\right) + m_2\left(\mathbf{u}_2^2 - \mathbf{u}_{2f}^2\right) = 0. \tag{2}$$

1. (5 pts) Use Matlab's symbolic toolbox to solve equations (1) and (2) for the final velocities, $\mathbf{u}_{1f}$ and $\mathbf{u}_{2f}$ in terms of the initial velocities $\mathbf{u}_1$, $\mathbf{u}_2$, and the object masses, m_1 and m_2. Use Matlab's `pretty` function to display the results. Note: you should obtain two roots. In your report, briefly interpret each.

2. (10 pts) Plot $\mathbf{u}_{1f}$ and $\mathbf{u}_{2f}$ as functions of $\frac{m_2}{m_1}$ for $\mathbf{u}_2 - \mathbf{u}_1 = \begin{bmatrix} -5 & -2 & 0 & 2 & 5 \end{bmatrix}$. Discuss your results. Note that for $\mathbf{u}_{1f}$ your plot should look like Figure 2.

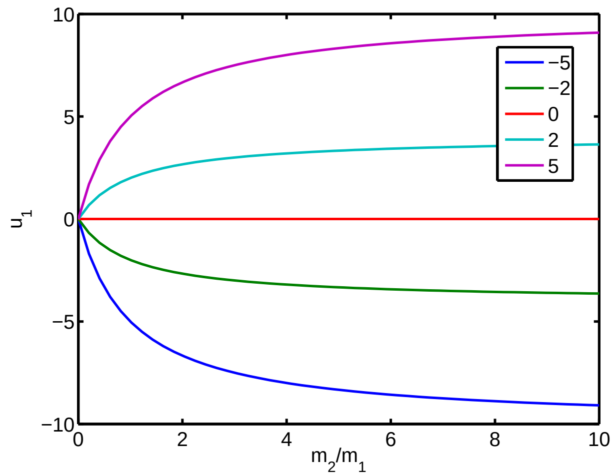

Figure 2: Results for $\mathbf{u}_{1f}$ as a function of $\frac{m_2}{m_1}$ for various values of $\mathbf{u}_2 - \mathbf{u}_1$.

Problem 3 (10 pts) - No report - submit M-file.

1. (5 pts) Use MATLAB to obtain the analytic solution for the roots of a general cubic polynomial, $y = ax^3 + bx^2 + cx + d$. Convert the answer you obtain to an executable MATLAB expression using the `vectorize` command. Implement a function to calculate the roots of a cubic function using the analytic solution.

2. (5 pts) The Van der Waals equation of state is of the form:

$$p = \frac{RT}{\bar{V} - b} - \frac{a}{\bar{V}^2}.$$

- Rearrange this into a cubic equation for the $\bar{V}$, in other words, write it in a form $\alpha\bar{V}^3 + \beta\bar{V}^2 + \delta\bar{V} + \gamma = 0$. Note that I couldn't get MATLAB to do this - I did it by hand. If you can get MATLAB to do this, I would like to know how you did it!
- Use the function you created in part 1 along with the Van der Waals equation (in cubic form) to estimate the molar volume of water at $T = 200\,\mathrm{C} = 473.15\,\mathrm{K}$ and $p = 2\,\mathrm{bar}$. The constants for water are $a = 5.536\,\frac{\mathrm{L^2 bar}}{\mathrm{mol^2}}$ and $b = 0.03049\,\frac{\mathrm{L}}{\mathrm{mol}}$. Note that the value of the gas constant in consistent units is $R = 0.083415\,\frac{\mathrm{L\,bar}}{\mathrm{mol\,K}}$.
- Verify your solution using the `roots` function in MATLAB.