Proper Orthogonal Decomposition Analysis of 9-Point Swirl-Venturi Lean Direct Injection Combustion Technology

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To reduce aircraft engine emissions of nitrogen oxides (NO\textsubscript{x}), NASA Glenn has developed experimental, multi-element lean direct injection (LDI) combustion technology. As the name implies, in LDI the combustor operates fuel-lean without a rich front end: all of the combustion air not used for liner cooling enters through the combustor dome. To prevent local near-stoichiometric regions that produce high NO\textsubscript{x} emissions, LDI relies on rapid and uniform fuel/air mixing. Therefore, understanding how and where mixing occurs in the flow field is critical to understanding the fuel/air mixing. So far, only time-averaged and root-mean-square (RMS) values have been used to study the fuel/air mixing. Experimentally measured RMS values are nearly as high as the mean values, i.e., the turbulence intensity is near 100%. However, this high turbulence intensity may be caused not by high levels of small-scale turbulent fluctuations but instead by coherent turbulent structures. Coherent structures have been found in time-accurate computational fluid dynamics (CFD) simulations of LDI. Therefore, in order to fully understand fuel/air mixing in LDI, a quantitative method for analyzing the observed turbulent fluctuations and coherent structures needs to be developed. A reasonable candidate for such an analysis tool is proper orthogonal decomposition (POD). This paper applies POD on one time series of LDI flame luminosity images and demonstrates that POD is indeed an appropriate technique for analyzing coherent structures in LDI.

1 Introduction

For more than 40 years, NASA has sustained programs to reduce the environmental effects of aviation. A major focus of these programs has been reducing the emissions of nitrogen oxides (NO\textsubscript{x}). NO\textsubscript{x} emissions decrease the protective ozone layer in the stratosphere and increase smog and ozone in the lower troposphere\cite{1}. To prevent damage to the protective ozone layer, NASA programs that target supersonic flight have focused on reducing cruise NO\textsubscript{x}. To reduce the emissions of NO\textsubscript{x} in the lower troposphere, NASA programs that target subsonic flight also focus on reducing NO\textsubscript{x} emissions in the landing-takeoff cycle.

In addition to decreasing NO\textsubscript{x} emissions, NASA focuses on reducing carbon dioxide emissions by increasing engine efficiency. Next generation aircraft gas turbines will need to increase fuel efficiency while simultaneously decreasing NO\textsubscript{x} emissions. Unfortunately, for a given combustor technology, increasing engine efficiency leads to increased NO\textsubscript{x} emissions. Therefore, improved low-NO\textsubscript{x} combustor technologies need to be developed.

In order to reduce NO\textsubscript{x} emissions, NASA has investigated several combustion concepts. These concepts fall into two categories: rich-front-end and lean front-end. Rich-front-end combustors
are similar traditional combustors in that the primary combustion zone is fuel-rich — only part of the combustion air enters through the fuel/air mixers; the rest enters through downstream dilution jets, allowing combustion to be completed. Rich-front-end combustion concepts include rich-burn–quick quench–lean-burn (RQL) and rich-front-end combustion[2, 3, 4]. In contrast, lean-front-end combustors operate fuel-lean throughout: All of the combustor air except that used for liner cooling enters through the combustor dome. Lean front end combustion concepts include lean, premixed, prevaporized (LPP), lean partially premixed, and lean direct injection (LDI) combustion[2, 5, 6, 7, 8]. This paper focuses on an LDI combustion concept.

LDI and other lean front end combustion concepts minimize local flame temperature. This keeps NO\textsubscript{x} emissions low because NO\textsubscript{x} is an exponential function of local temperature. To minimize NO\textsubscript{x} emissions, fuel-lean combustion needs to avoid local near-stoichiometric zones where the flame temperature is high. Avoiding these zones requires rapid and uniform fuel/air mixing. LDI accomplishes this mixing in part by replacing one traditionally-sized fuel/air mixer with multiple smaller fuel/air mixers. But previous experiments have shown that decreasing the size and increasing the number of fuel/air mixers is not sufficient to improve mixing: Fuel/air mixer design is also important.

Multiple fuel/air mixer designs have been studied. There are several ways an individual fuel/air mixer (the fuel injector) can be constructed. On the air side, a radial, axial, or discrete jet swirler may be used. With any of these types of swirlers, the swirl number can be varied. A venturi can be placed downstream of the air swirler, or the venturi can be omitted. On the fuel side, a simplex or air assist-atomizer can be used. For a simplex atomizer, flow number (i.e., effective flow area) can be varied. In addition, once a fuel/air mixer design has been chosen, the size and number of the mixer elements can be varied.

In order to choose the best fuel/air mixer design for an LDI combustor, it is critical to understand the fuel/air mixing process. Understanding fuel/air mixing requires knowledge both of large overall features such as recirculation zones and precessing vortices and of smaller eddies where much of the mixing occurs. Some large structures have been identified in simulations and experiments. For example, computational fluid mechanics (CFD) simulations have indicated the presence of coherent structures, such as precessing vortices. Coherent structures can also be picked out by eye in high-speed images of LDI flames. However, we need a way to systematically examine and compare both the large overall features and the smaller eddies. This paper explores a way to do this using Proper Orthogonal Decomposition (POD), a widely used technique for finding structures in complex data. In this paper, POD will be applied to high-speed flame luminescence images from the baseline LDI geometry.

2 Experimental Facilities and Hardware

2.1 Experimental Facilities and Optical Diagnostics

The LDI experiments described in this paper were done in NASA Glenn’s CE-5 flametube combustion test rig[9]; see the drawing in Figure 1. CE-5 can supply nonvitiated air preheated to 867 K at pressures up to 1.7 MPa (3.1 MPa if windows are not used). The CE-5 flametube was configured to have a test section with a 76.2-mm × 76.2-mm square cross section. This test section has 4
nitrogen-cooled quartz windows spaced 90° degrees apart; each window is 38.1-mm by 50.8-mm. These quartz windows were used to obtain high-speed images of the flame chemical luminescence. The high-speed images were taken with a 12-bit, high-speed CMOS camera with a 1024×1024-pixel array. This array was sensitive to visible light; therefore, the light collected was largely a combination of CH and C₂ emissions. The camera was fitted with a f=150-mm, f/1.2 lens and was focused on the vertical center plane. Light was collected from the entire field-of-view: the measurements are line-of-sight[10].

For the flame luminosity measurements analyzed in this paper, the camera frame rate was 10 kHz and the image size was 944×576.

2.2 LDI Configurations

This paper describes results for the baseline 9-point swirl venturi lean direct injection (SV-LDI) geometry[6]; see the photograph in Figure 2. The 9-point SV-LDI geometry consists of 9 fuel/air mixers arranged in a 3×3 grid with a 76.2-mm × 76.2-mm square cross section. The center-to-center distance between fuel/air mixers is 25.4 mm.

Each fuel/air mixer consists of a simplex fuel injector and an air passage with a six-bladed, helical axial air swirler followed by a converging-diverging venturi section (see Figure 3). The diameter of the venturi throat is 1.27 cm. The simplex fuel injector is inserted through the center of the air swirler, with the fuel injector tip at the venturi throat. The per injector flow number FN_{US} (as defined by Lefebvre[11]) is 2.9. All swirlers are co-rotating with a blade angle of 60°. Each blade has an inside diameter of 9.4 mm and an outside diameter of 22 mm. The calculated swirl number, as defined by Beer and Chigier[12], is 1.02. The measured effective area of the air swirler array is 1000 mm².
2.3 Test Conditions

This paper analyzes high speed flame luminosity images from one SV-LDI test point. At this test point, the combustor inlet pressure $p_3$ was 1034 kPa, the inlet temperature $T_3$ was 828 K, the mass flow rate $\dot{m}$ was 0.60 kg/s, and the fuel-to-air ratio, $f$, was 0.024. These conditions resulted in a pressure drop across the combustor $\Delta p_{3-4}/p_3$ of 4% and a calculated adiabatic flame temperature $T_{ad}$ of 2461 K.

The bulk velocity in the flametube can be calculated two ways: using the combustor inlet conditions $T_3$ and $p_3$ and using the burned gas conditions $p_4$ and $T_{ad}$. The bulk velocity $u_3$ based on combustor inlet conditions is 23.7 m/s, and the bulk velocity $u_4$ based on burned gas conditions is 48.4 m/s.

3 Analysis

3.1 Expected Length and Time Scales

Both the length and time scales vary by several orders of magnitude in turbulent flows. The largest scales are those associated with the physical geometry of the flow. The largest length scales will be referred to as the “outer scale,” $l$. For the 9-point SV-LDI geometry, the relevant physical length scales include the flametube height and width (both 7.62-cm), the center-to-center distance between fuel/air mixers (2.54-cm), and the diameter of the venturi throat (1.27-cm).

This outer length scale can be used to define an outer time scale, $t$. Following Tennekes and Lumley[13], this outer time scale can be estimated by dividing the outer length scale by a relevant velocity, $u$, i.e., $t = l/u$. For the SV-LDI flametube tests, the relevant velocity will be either the combustor inlet bulk velocity $u_3$ or the burned gas bulk velocity $u_4$.

Note that outer length and time scales are estimates. The size of the largest features observed in the flow field should be roughly on the same order of magnitude as the outer length scale, and these
large-scale features should evolve at a rate of the same order of magnitude as the outer time scale.

The scale of the features observed in turbulent flow fields varies from the largest scales discussed above down to the smallest scale that can be sustained by a turbulent flow field: the Kolmogorov scale. The Kolmogorov scale is the length scale at which viscosity begins to dominate the flow. It can be calculated as \( \eta = lRe^{-\frac{3}{4}} \), where \( \eta \) is the Kolmogorov length scale, \( l \) is the outer length scale and \( Re \) is the Reynolds number based on outer-scale values. Similarly, the Kolmogorov time scale \( \tau \) can be calculated as \( \tau = tRe^{-\frac{1}{2}} \), where \( t \) is the outer time scale. Again, as with the outer scales, the Kolmogorov length and time scales are order-of-magnitude estimates.

This paper will focus on identifying features of the size of the outer length scale.

### 3.2 Proper Orthogonal Decomposition

The flowfield features will be identified using proper orthogonal decomposition (POD). Proper orthogonal decomposition is a technique used to extract large-scale features from measurements or simulations[14, 15, 16].

Using the language of linear algebra, POD creates an orthogonal basis that completely describes the data set; this data basis is optimal in that most of the fluctuations in the data set are captured in the first few basis functions (“modes”) of the basis. Although a data set may have hundreds or even thousands of modes, POD can accurately model the data set with only a handful of modes.

The POD is normally calculated in one of two ways: either by using the ”direct method” using a singular value decomposition (svd) or by using the ”method of snapshots.” The direct method tends to be more appropriate for small data sets, and the method of snapshots for large data sets. For this paper, the POD will be calculated using the method of snapshots[14].

### 4 Results and Discussion

#### 4.1 Relevant Length and Time Scales

As discussed in section 3.1, the outer length scale \( l \) should be on the order of one of the 9-point SV-LDI geometric features: the flametube width/height, the fuel/air mixer center-to-center distance, and the venturi throat diameter. Which of these possible outer length scales is most applicable can be best determined from actual measurements.

High speed flame luminosity measurements are shown in Figure 4–5. Figure 4 shows 20 sequential images and 5 shows the mean and root-mean-square (RMS) calculated from 8055 images. Inspection of both Figure 4 and Figure 5 indicates that the dominant length scale is on the order of the center-to-center distance (25.4 mm). The instantaneous images in Figure 4 show features with axial widths on the order of 10 mm and vertical heights on the order of 20 mm. For example, in image 1, there is a region of high luminosity extending axially from 0 to about 15 mm and vertically from about 25 to about 45 mm. In image 2, one region of high luminosity extends axially from about 3 to about 15 mm and vertically from about 25 to about 40 mm. Similarly-sized features can be seen in the other 18 images. The mean RMS luminosity shown in Figure 5 show similarly-sized
features. The largest region of high mean flame luminosity extends axially from 3-15 mm and vertically from 20-45 mm. A large region of very high (above 400) RMS values extends axially from 3-17 mm and vertically from 22-45 mm; a region of elevated RMS values (above 300) extends axially from 3-25 mm and vertically the entire height of the window (5-61 mm). When choosing a reference length scale, it is reasonable to choose the larger of the axial and vertical feature sizes. The features seen in the instantaneous, mean, and RMS images usually extend farther vertically than they do axially, and the most common vertical extent is about 20 mm. This is close to the geometrically-relevant center-distance of 25.4 mm. Therefore, the reference outer length scale \( l \) will be set as 25.4 mm for the remainder of this paper.

The reference outer length scale is used along with the reference outer velocity scale to determine the reference time scale. However, determining the reference outer velocity scale is not as straightforward as determining the reference length scale. An obvious candidate for the reference velocity is the burned gas bulk velocity. However, instantaneous particle image velocimetry (PIV) measurements show that bulk burnt gas velocity underestimates the actual velocities seen in the flow. Due to the observed recirculation zones just downstream of each fuel/air mixer, the maximum local mean velocities will be larger than the bulk burnt gas velocity. In addition, measurements show that RMS velocity fluctuations can be almost as large as the mean velocity. This means that the characteristic reference velocity should be at least a couple times larger than the bulk burnt gas velocity. How much larger it should be is hard to determine. Based on the data presented it Heath et al A reasonable estimate is 5-10 times larger than the bulk burnt gas velocity. For this paper, the reference outer velocity scale will be set to 10 times the bulk burnt gas velocity; this yields a reference outer velocity scale \( u \) of 484 m/s.

With the reference outer length and velocity scales determined, the reference outer time scale \( t = l/u \) can be calculated; it is 52.5 \( \mu \)s. Taking \( 1/t \) yields a reference frequency of 19 kHz. To take fully time-resolved measurements should require a measurement rate of several times the reference frequency: the frequency needed for time-resolution is expected to be on the order of 50-200 kHz.

The instantaneous flame luminosity measurements can be used to verify that the calculated reference outer time scale is reasonable. If the measurements were time-resolved or nearly time-resolved, one would expect to be able to follow flow features as they move downstream from one frame to the next; however, if the measurements were not time-resolved, one would expect that it would be difficult or impossible to follow features from frame to frame. To be reasonably well time-resolved, the frame rate should be several times the reference outer time scale. The flame luminosity measurements shown in Figure 4 were taken at a frame rate of 10 kHz, so there is 0.1 ms = 100 \( \mu \)s between successive images; this is almost double the outer time scale of 52.5 \( \mu \)s. If this outer time scale estimate is correct, one would expect not to be able to follow features from frame to frame; examining the 20 successive images in Figure 4 verifies that we cannot follow flow features from one frame to the next.

The calculated reference outer time scale can be verified more quantitatively using the autocorrelation function. The autocorrelation function is defined as:

\[
a(x, y) = \frac{\langle I'(x, y, t)I'(x, y, t + n\Delta t) \rangle}{I^2_{\text{RMS}}(x, y)}
\]

(1)

\[
I'(x, y, t) = I(x, y, t) - \bar{I}(x, y),
\]

(2)
Figure 4: Twenty successive flame luminosity images. The time between images is 0.1 ms. Flow is left to right.
Figure 5: Mean (left) and root-mean-square (RMS, right) of 8055 flame luminosity images. Flow is left to right.

where \( a(x, y) \) is the autocorrelation function at point \((x, y)\), \( I(x, y, t) \) is the image intensity at point \((x, y)\) and time \(t\), and \( \bar{I}(x, y) \) is the mean image intensity at point \((x, y)\). If the measurements are time-resolved, the autocorrelation function should remain near unity for several image frames. If the measurements are not time-resolved, the autocorrelation should quickly fall to zero.

The autocorrelation function at four representative points in the flame zone is shown in Figure 6. On the left, the mean luminosity from Figure 5 is reproduced; marks indicate the locations used for the autocorrelation. The autocorrelation is on the right. At all four points, the autocorrelation drops to below 40% in one frame and below 20% in two frames. This confirms that the flame luminosity images are not time-resolved.

4.2 Proper Orthogonal Decomposition

As stated above, POD creates an orthonormal basis that contains most of the fluctuations in the first few modes. Therefore, if POD is an appropriate technique for extracting features from flame luminosity images, it should be able to capture most of the luminosity fluctuations in a small number of modes. In addition, if coherent structures dominate the flow field, the first few modes should contain large features; few small-scale fluctuations should be seen. Instead, the small-scale features should appear in the POD modes that capture only a small percentage of the luminosity fluctuations.

To test this, a POD analysis was done on the fluctuations in flame luminosity, i.e., on \( I' = I - \bar{I} \). Although thousands of spatial modes are possible, the POD should be able to capture most the fluctuations in only a small number of modes. Therefore, only 400 POD modes were calculated.
**Figure 6**: Measured autocorrelation function. The image on the left duplicates the mean flame luminosity shown in Figure 5, with added markers showing the four locations chosen for the autocorrelation function. The plot of the right shows the autocorrelation function at these four locations. For convenience, the $x$-axis shows image separation. Since the frame rate was 10 kHz, multiplying the Image Separation by 0.1 will give the time in ms.

The results are shown in Figure 7 – 9. (Note that the POD modes shown in Figures 8 and 9 are normalized so that they can be compared more easily.)

Figure 7 shows that the first few POD modes do indeed capture most of the fluctuations. The percentage of fluctuations captured by a given mode is shown in the left-hand plot, and the cumulative sum of fluctuations captured is shown in the right-hand plot. (The cumulative total is calculated in the sense of, equivalently, least-squares, the $L_2$ norm, or energy.) Although 400 modes were calculated, the percentage of fluctuations captured by each mode drops rapidly after the first few modes, so only the first 50 modes are shown. Figure 7 shows that the first mode explains 71% of the fluctuations, the second mode 36%, and the third through tenth modes between 27% and 11% each. As a result, the first ten POD modes capture more than 94% of the flame luminosity fluctuations. The fact that the first several POD modes capture most of the image fluctuations shows that POD can accurately model the flame luminosity images using a small number of terms.

The first ten POD modes are shown in Figure 8. These modes do indeed contain large scale features. For example, the first mode contains a large feature that extends about 10 mm in the axial direction and 20 mm in the vertical direction. Even though the feature size gradually decreases as the mode number increases, the features seen in the first ten modes are still on the order of the reference outer length scale of 25.4 mm. For example, mode 10 contains features on the order of 10-mm by 10-mm in size.

In contrast, the later modes contain many more small features. Figure 9 contains four of the later POD modes: the 100th, the 200th, the 300th, and the 400th mode. Modes 100 contains many features about 1-2 mm in size and modes 200 and 300 contain many features less than 1 mm in size, much smaller than than 10- to 20-mm-sized features seen in the first ten modes. (Mode 400 contains only very weak fluctuations.) These modes each capture only a small percentage of the flame luminosity fluctuations: 0.66%, 0.16%, 0.06%, and 0.01%, respectively.
Figure 7: Mode strength. The plot on the left shows the percentage of fluctuations captured by a given mode, while the plot on the right shows the cumulative sum of fluctuations captured.

Figure 8: The first ten POD modes.
Figure 9: Higher POD modes that capture only a small percentage of the fluctuations in flame luminosity. From left to right, the 100th, 200th, 300th, and 400th POD modes. The 100th mode captures 0.66% of the flame luminosity fluctuations, the 200th mode 0.16%, the 300th mode 0.06%, and the 400th mode 0.01%.

5 Summary

Fully understanding fuel/air mixing in LDI combustion requires a quantitative method for analyzing coherent structures in experimental measurements and CFD simulations. This paper reports progress in developing such a method. First, the outer scale length- and time-scales were estimated based on previously measured data. These estimates yielded an outer length scale on the order of 25.4 mm and an outer time scale on the order of 52.5 \( \mu s \). The reference time scale yields a reference frequency of 19 kHz; to take fully time-resolved measurements, a measurement rate of several times the reference frequency is required. The reference length scale was verified to be on the order of 25.4 mm by examining flame luminosity measurements. These flame luminosity measurements were taken at a frame rate of 10 kHz, so the time between frames was 100 \( \mu s \). This is almost twice the reference outer time scale, so it is expected that these measurements are not time resolved; this was verified by observing that one cannot follow the flow by eye and by calculating the autocorrelation function. Then, a POD analysis was performed on this time series of flame luminosity measurements. The POD analysis showed that more than 94% of the fluctuations in flame luminosity can be explained by the first ten POD modes. The first ten POD modes all had length scales on the order of the outer length scale, as would be expected if the first POD modes were capturing coherent structures. This demonstrates that POD is indeed an appropriate technique for analyzing coherent structures in LDI.

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