Rate-Ratio Asymptotic Analysis of the Influence of Addition of Hydrogen on the Structure and Mechanisms of Extinction of Nonpremixed Methane Flame

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Rate-ratio asymptotic analysis is carried out to elucidate the influence of hydrogen on the structure and mechanisms of extinction of methane flames in laminar nonpremixed flows. Steady, axisymmetric, laminar flow of two counterflowing streams toward a stagnation plane is considered. One stream, called the fuel stream, is made up of a mixture of methane, hydrogen, and nitrogen. The other stream, called the oxidizer stream is made up of a mixture of oxygen, and nitrogen. A reduced four-step chemical kinetic mechanism is employed. Chemical reactions are presumed to take place in a thin reaction zone that is established in the vicinity of the stagnation plane. On either side of this thin reaction zone, the flow field is inert. These inert regions are called the outer structure. The outer structure is analyzed first. It gives the matching conditions that is required in the analysis of the reaction zone. In the thin reaction zone chemical reactions are presumed to take place in two layers—an inner layer and an oxidation layer. In the inner layer methane is consumed and hydrogen and carbon monoxide are formed. In the oxidation layer oxygen, carbon monoxide and hydrogen are consumed. Critical conditions of extinction are predicted and are compared with experimental data and with results of numerical computation.

1 Introduction

Recently an experimental and computational study of was completed to elucidate the influence of hydrogen on the structure and critical conditions of extinction of laminar nonpremixed methane flames [1]. In this study hydrogen was added to either methane-nitrogen mixtures or oxygen-nitrogen mixtures at normal atmospheric pressure, with both feed streams at normal room temperature. Experimental conditions were adjusted to fix selected values of the stoichiometric mixture fraction and the adiabatic flame temperature, and the strain rate was increased gradually, maintaining the momentum balance of the two streams, until extinction occurred. At the selected sets of values, the strain rate at extinction was measured as a function of the hydrogen concentration in the fuel or oxidizer stream [1]. The ratio of the fraction of the oxidizer flux that consumes hydrogen to the fraction that consumes fuel was calculated, and it was found that, within experimental uncertainty, the ratio of the extinction strain rate with hydrogen addition to that without was the same at any given value of this oxygen flux ratio, irrespective of whether the hydrogen was added on the fuel or oxidizer side. This experimental result was also in close agreement with computational predictions employing detailed chemistry. Here a rate ratio asymptotic analysis is carried out.
to obtain an improved fundamental understanding of the influence of hydrogen on nonpremixed methane flames.

A number of rate-ratio asymptotic analyses of the structure of nonpremixed methane flames are available [2–10]. The present analysis employs the formulation described by Bai and Seshadri [10]. In this previous formulation the asymptotic structure of laminar, nonpremixed methane flames was analyzed using a reduced four-step chemical-kinetic mechanism [10]. The asymptotic flame structure was presumed to be made up of a thin reaction zone, where chemical reactions take place. The outer zones on either side of this reaction zone were chemically inert. The results of the asymptotic analysis were used to predict the scalar dissipation rates extinction [10]. This previous analysis is extended here to elucidate the influence of hydrogen addition to the reactant streams on the scalar dissipation rate at extinction.

2 Reduced Mechanism

The elementary chemical-kinetic mechanism from which the reduced mechanism is derived is made up of reactions 1–35 and 37–40 shown in Table 1.1 of Ref. [11]. Steady-state approximations are introduced for CH$_3$, CH$_2$, CH, CH$_2$O, CHO, HO$_2$, H$_2$O$_2$, O, and OH. The reduced four-step mechanism can be written as

\[
\begin{align*}
\text{CH}_4 + 2\text{H} + \text{H}_2\text{O} & \rightleftharpoons \text{CO} + 4\text{H}_2, & \text{I} \\
\text{CO} + \text{H}_2\text{O} & \rightleftharpoons \text{CO}_2 + \text{H}_2, & \text{II} \\
\text{H} + \text{H} + \text{M} & \rightleftharpoons \text{H}_2 + \text{M}, & \text{III} \\
\text{O}_2 + 3 \text{H}_2 & \rightleftharpoons 2 \text{H} + 2 \text{H}_2\text{O}. & \text{IV}
\end{align*}
\]

Global reaction I is chain-breaking and represents the reactions between the fuel and the radicals which form CO and H$_2$. Global reaction II represents the oxidation of CO to form the final product, CO$_2$. Global reaction III represents the three-body recombination steps and is also responsible for a major fraction of heat released in the flame. Global reaction IV represents the reaction of O$_2$ with the radicals and the formation of H$_2$O; it comprises the chain-branching steps. Table 1 shows the elementary reactions which are presumed to be the major contributors to the rates of the global steps of the reduced mechanism. The symbols $f$ and $b$ appearing in the first column of Table 1, respectively, identify the forward and backward steps of a reversible elementary reaction $n$. The symbol M represents a third body. The asymptotic analysis uses only the elementary reactions shown in Table 1.

The reaction rates of the global steps $w_k$ in the four-step mechanism ($k = \text{I–IV}$), expressed in terms of the reaction rates of elementary reactions $w_n$, are

\[
\begin{align*}
 w_I &= w_{7f} - w_{7b} - w_8, \\
 w_{II} &= w_{6f} - w_{6b}, \\
 w_{III} &= w_5 + w_8, \\
 w_{IV} &= w_{1f} - w_{1b}.
\end{align*}
\]

The steady-state concentrations of O, OH and CH$_3$ are required for calculating the global rates of the reduced mechanism. To simplify the calculations, the elementary reactions 2 and 3 are
Table 1: Rate data for elementary reactions employed in the asymptotic analysis.

<table>
<thead>
<tr>
<th>Number</th>
<th>Reaction</th>
<th>$B_n$</th>
<th>$\alpha_n$</th>
<th>$E_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1f</td>
<td>$\text{O}_2 + \text{H} \rightarrow \text{OH} + \text{O}$</td>
<td>2.000E+14</td>
<td>0.00</td>
<td>70.30</td>
</tr>
<tr>
<td>1b</td>
<td>$\text{O} + \text{OH} \rightarrow \text{H} + \text{O}_2$</td>
<td>1.568E+13</td>
<td>0.00</td>
<td>3.52</td>
</tr>
<tr>
<td>2f</td>
<td>$\text{H}_2 + \text{O} \rightarrow \text{OH} + \text{H}$</td>
<td>5.060E+04</td>
<td>2.67</td>
<td>26.3</td>
</tr>
<tr>
<td>2b</td>
<td>$\text{H} + \text{OH} \rightarrow \text{O} + \text{H}_2$</td>
<td>2.222E+04</td>
<td>2.67</td>
<td>18.29</td>
</tr>
<tr>
<td>3f</td>
<td>$\text{H}_2 + \text{OH} \rightarrow \text{H}_2\text{O} + \text{H}$</td>
<td>1.000E+08</td>
<td>1.60</td>
<td>13.80</td>
</tr>
<tr>
<td>3b</td>
<td>$\text{H} + \text{H}_2\text{O} \rightarrow \text{OH} + \text{H}_2$</td>
<td>4.312E+08</td>
<td>1.60</td>
<td>76.46</td>
</tr>
<tr>
<td>4f</td>
<td>$\text{OH} + \text{OH} \rightarrow \text{H}_2\text{O} + \text{O}$</td>
<td>1.500E+09</td>
<td>1.14</td>
<td>0.42</td>
</tr>
<tr>
<td>4b</td>
<td>$\text{O} + \text{H}_2\text{O} \rightarrow \text{OH} + \text{OH}$</td>
<td>1.473E+10</td>
<td>1.14</td>
<td>71.09</td>
</tr>
<tr>
<td>5a</td>
<td>$\text{H} + \text{O}_2 + \text{M} \rightarrow \text{HO}_2 + \text{M}$</td>
<td>2.300E+18</td>
<td>-0.80</td>
<td>0.00</td>
</tr>
<tr>
<td>6f</td>
<td>$\text{CO} + \text{OH} \rightarrow \text{CO}_2 + \text{H}$</td>
<td>4.400E+06</td>
<td>1.50</td>
<td>-3.10</td>
</tr>
<tr>
<td>6b</td>
<td>$\text{H} + \text{CO}_2 \rightarrow \text{OH} + \text{CO}$</td>
<td>4.956E+08</td>
<td>1.50</td>
<td>89.76</td>
</tr>
<tr>
<td>7f</td>
<td>$\text{CH}_4 + \text{H} \rightarrow \text{CH}_3 + \text{H}_2$</td>
<td>2.200E+04</td>
<td>3.00</td>
<td>36.60</td>
</tr>
<tr>
<td>7b</td>
<td>$\text{CH}_3 + \text{H}_2 \rightarrow \text{CH}_4 + \text{H}$</td>
<td>8.391E+02</td>
<td>3.00</td>
<td>34.56</td>
</tr>
<tr>
<td>8b</td>
<td>$\text{CH}_3 + \text{H} \rightarrow \text{CH}<em>4 + k_0/ k</em>\infty$</td>
<td>6.257E+23</td>
<td>-1.80</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.108E+14</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>$\text{CH}_3 + \text{O} \rightarrow \text{CH}_2\text{O} + \text{H}$</td>
<td>7.000E+013</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Units are moles, cubic centimeters, seconds, kJoules, Kelvin.

- Third body collision efficiencies are $[\text{M}]=6.5[\text{CH}_4]+1.5[\text{CO}_2]+0.75[\text{CO}]+0.4[\text{N}_2]+6.5[\text{H}_2\text{O}]+0.4[\text{O}_2]+1.0[\text{Other}]$.

- For reaction 8: $k = F k_\infty k_0 [p/(\tilde{R}T)] / \{k_\infty + k_0 [p/(\tilde{R}T)]\}$, where $\log_{10} F = \log_{10} F_c / \{1 + (\log_{10} (k_0 [p/(\tilde{R}T)]/k_\infty))^{1/2} \tilde{N}\}^2 \N = 0.75 - 1.27 \log_{10} F_c$, and $F_c = 0.577 \exp[-T/2370.0]$.
presumed to maintain partial equilibrium.

\[ C_{\text{OH}} = \frac{C_{\text{H}_2\text{O}} C_{\text{H}}}{K_3 C_{\text{H}_2}}, \quad C_{\text{O}} = \frac{C_{\text{H}_2\text{O}} C_{\text{H}}^2}{K_2 K_3 C_{\text{H}_2}^2}, \]  

(2)

where \( C_i \) is the molar concentration of species \( i \), and \( K_n \) is the equilibrium constant of the elementary step \( n \). From the chemical-kinetic mechanism shown in Table 1 it follows that \( K_4 = K_3/K_2 \).

3 Formulation

Figure 1 is a schematic representation of the flowfield analyzed here. Here two counterflowing streams flowing toward a stagnation plane. On stream called fuel stream is made up of CH\(_4\) and N\(_2\) and the other stream, called the oxidizer stream, is made up of O\(_2\) and N\(_2\). Hydrogen is added to the fuel stream or to the oxidizer stream. The mass fraction of CH\(_4\) in the fuel stream is \( Y_{\text{F},1} \), and that of O\(_2\) in the oxidizer stream is \( Y_{\text{O},2} \). When H\(_2\) is added to the fuel stream its mass fraction in the fuel stream is \( Y_{\text{H}_2,1} \) and it is \( Y_{\text{H}_2,2} \) when added to the oxidizer stream.

In view of the high diffusivity of hydrogen in comparison to the other reactants, it is convenient to introduce the conserved scalar quantities \( \xi \) and \( \xi_{\text{H}_2} \) defined by the equations

\[
x \frac{d \xi}{dx} + \frac{d^2 \xi}{dx^2} = 0
\]

\[
x \frac{d \xi_{\text{H}_2}}{dx} + \frac{1}{L \xi_{\text{H}_2}} \frac{d^2 \xi_{\text{H}_2}}{dx^2} = 0
\]

(3)
Here $x$ is a spatial co-ordinate normal to the flame sheet made nondimensional with respect to the thermal diffusivity and the strain rate, $\text{Le}_i = \lambda / (\rho c_p D_i)$ is the Lewis number of species $i$, $\rho$ is the density, $\lambda$ the coefficient of thermal conductivity, $c_p$ the heat capacity, and $D_i$ the coefficient of diffusion of species $i$. The stagnation plane is at $x = 0$, the oxidizer stream is at $x \rightarrow \infty$ and the fuel stream at $x \rightarrow - \infty$. Equation (3) is constrained to satisfy the conditions

$$
\begin{align*}
\xi &= \xi_{H_2} = 0; \quad x = \infty \\
\xi &= \xi_{H_2} = 1; \quad x = -\infty
\end{align*}
$$

Integration of Eq. (3) together with Eq. (4) gives

$$
\begin{align*}
\xi &= \frac{1}{2} \text{erfc} \left( x \sqrt{\frac{1}{2}} \right) \\
\xi_{H_2} &= \frac{1}{2} \text{erfc} \left( x \sqrt{\frac{\text{Le}_{H_2}}{2}} \right).
\end{align*}
$$

It follows from Eq. (5)

$$
\frac{d\xi_{H_2}}{dx} = \left( \sqrt{\text{Le}_{H_2}} \right) \frac{d\xi}{dx} \left\{ \exp \left[ \frac{x^2 \left( 1 - \text{Le}_{H_2} \right)}{2} \right] \right\}
$$

For convenience, the definitions

$$
\begin{align*}
X_i &= Y_i W_{N_2} / W_i, \\
\tau &= (T - T_{\text{ref}}) / \Delta T_{\text{ref}}, \\
G_F &= Q_F / (W_{N_2} c_p \Delta T_{\text{ref}}), \\
G_{H_2} &= Q_{H_2} / (W_{N_2} c_p \Delta T_{\text{ref}})
\end{align*}
$$

are introduced. Here, $Q_F$ is the heat released per mole of CH$_4$ consumed and $Q_{H_2}$ is the heat released per mole of H$_2$ consumed, $W_i$ is the molecular weight of species $i$, $W_{N_2}$, is the molecular weight of nitrogen, $T_{\text{ref}}$ is a reference temperature and $\Delta T_{\text{ref}}$ is a reference temperature difference. Figure 2 shows the outer structure in the limit of large values of the Damköhler number. Let the flame sheet be located at $x = x_{\text{st}}$. At $x_{\text{st}}$ there is complete consumption of CH$_4$, H$_2$, and O$_2$. At $x_{\text{st}+}$ the gradients are

$$
\begin{align*}
\frac{dX_F}{dx} &= \frac{dX_F}{d\xi} \frac{d\xi}{dx} = \frac{X_{F,1}}{1 - \xi_{\text{st}}} \frac{d\xi}{dx} \\
\frac{dX_{O_2}}{dx} &= 0 \\
\frac{1}{\text{Le}_{H_2}} \frac{dX_{H_2}}{dx} &= \frac{1}{\text{Le}_{H_2}} \frac{dX_{H_2}}{d\xi_{H_2}} \frac{d\xi_{H_2}}{dx} = \frac{1}{\text{Le}_{H_2}} \frac{X_{H_2,1}}{1 - \xi_{H_2,\text{st}}} \frac{d\xi_{H_2}}{dx} \\
\frac{dX_{CO_2}}{dx} &= \frac{dX_{CO_2}}{d\xi} \frac{d\xi}{dx} = -\frac{X_{CO_{2,\text{st}}}}{1 - \xi_{\text{st}}} \frac{d\xi}{dx} \\
\frac{dX_{H_2O}}{dx} &= \frac{dX_{H_2O}}{d\xi} \frac{d\xi}{dx} = -\frac{X_{H_2O_{\text{st}}}}{1 - \xi_{\text{st}}} \frac{d\xi}{dx} \\
\frac{d\tau}{dx} &= \frac{d\tau}{d\xi} \frac{d\xi}{dx} = -\frac{\tau_{\text{st}}}{1 - \xi_{\text{st}}} \frac{d\xi}{dx}
\end{align*}
$$

(8)
At \( x_{st} \) the gradients are

\[
\begin{align*}
\frac{dX_F}{dx} &= 0 \\
\frac{dX_O_2}{dx} &= \frac{X_{O_2,2}}{\xi_{st}} \frac{d\xi}{dx} \\
\frac{1}{Le_{H_2}} \frac{dX_{H_2}}{dx} &= \frac{X_{H_2,2}}{\xi_{st}} \frac{d\xi_{H_2}}{dx} \\
\frac{dX_{CO_2}}{dx} &= \frac{X_{CO_2,2}}{\xi_{st}} \frac{d\xi}{dx} \\
\frac{dX_{H_2O}}{dx} &= \frac{X_{H_2O,2}}{\xi_{st}} \frac{d\xi}{dx} \\
\frac{d\tau}{dx} &= \frac{\tau_{st}}{\xi_{st}} \frac{d\xi}{dx}
\end{align*}
\]

(9)

Balance equation for carbon across the reaction zone at \( x = x_{st} \) is

\[
\left( \frac{dX_F}{dx} + \frac{dX_{CO_2}}{dx} \right) = \left( \frac{dX_F}{dx} + \frac{dX_{CO_2}}{dx} \right)
\]

(10)

This gives

\[
X_{CO_2,2} = X_{F,1}\xi_{st}
\]

(11)

Balance equation for oxygen across the reaction zone at \( x = x_{st} \) is

\[
\left( 2\frac{dX_{O_2}}{dx} + 2\frac{dX_{CO_2}}{dx} + \frac{dX_{H_2O}}{dx} \right) = \left( 2\frac{dX_{O_2}}{dx} + 2\frac{dX_{CO_2}}{dx} + \frac{dX_{H_2O}}{dx} \right)
\]

(12)

This gives

\[
X_{H_2O,2} = 2X_{O_2,2} (1 - \xi_{st}) - 2X_{F,1}\xi_{st}
\]

(13)

Balance equation for hydrogen across the reaction zone at \( x = x_{st} \) is

\[
\left( 2\frac{dX_F}{dx} + \frac{1}{Le_{H_2}} \frac{dX_{H_2}}{dx} + \frac{dX_{H_2O}}{dx} \right) = \left( 2\frac{dX_F}{dx} + \frac{1}{Le_{H_2}} \frac{dX_{H_2}}{dx} + \frac{dX_{H_2O}}{dx} \right)
\]

(14)
This gives

\[
[X_{H_2,2} + \xi_{H_2,\text{st}} (X_{H_2,1} - X_{H_2,2})] \left( \frac{1 - \xi_{\text{st}}}{\xi_{H_2,\text{st}} (1 - \xi_{H_2,\text{st}})} \right) \frac{1}{\sqrt{Le_{H_2}}} \exp \left[ \frac{x_{\text{st}}^2 (1 - L e_{H_2})}{2} \right]
\]  

(15)

where the quantity \( r \) is

\[
r = \frac{X_{O_2,2}}{2X_{F,1}} \left( \frac{1 - \xi_{\text{st}}}{\xi_{\text{st}}} \right) - 1
\]  

(16)

Equations (5) and (15) are used to evaluate \( x_{\text{st}}, \xi_{\text{st}}, \) and \( \xi_{H_2,\text{st}} \) given \( X_{F,1}, X_{O_2,2}, X_{H_2,1}, \) and \( X_{H_2,2} \). Coupling relations for temperature gives

\[
\left[ \frac{d\tau}{dx} + 2G_{H_2} \frac{dX_{O_2}}{dx} + (G_F - 4G_{H_2}) \frac{dX_F}{dx} \right] = \left[ \frac{d\tau}{dx} + 2G_{H_2} \frac{dX_{O_2}}{dx} + (G_F - 4G_{H_2}) \frac{dX_F}{dx} \right]
\]  

(17)

This gives

\[
\tau_{\text{st}} = (G_F - 4G_{H_2}) X_{F,1} \xi_{\text{st}} + 2G_{H_2} X_{O_2,2} (1 - \xi_{\text{st}})
\]  

(18)

The adiabatic flame temperature \( T_{\text{st}} \) is

\[
T_{\text{st}} - T_u = \frac{Y_{F,1} (Q_F - 4Q_{H_2}) \xi_{\text{st}}}{c_p W_F} + \frac{2 \xi_{H_2,1}}{c_p W_{O_2}} (1 - \xi_{\text{st}})
\]  

(19)

Equation (19) is written as

\[
T_{\text{st}} - T_u = \frac{Y_{F,1} \xi_{\text{st}}}{c_p W_F} (Q_F + 4r Q_{H_2})
\]  

(20)

The reference temperature difference \( \Delta T_{\text{ref}} \) is chosen as

\[
\Delta T_{\text{ref}} = \frac{Q_F}{c_p W_{N_2}} = (T_{\text{st}} - T_u) \left[ \frac{Y_{F,1} W_{N_2} \xi_{\text{st}}}{W_F} \left( 1 - \frac{4Q_{H_2}}{Q_F} \right) \right] + \frac{2 \xi_{H_2,2} W_{N_2} (1 - \xi_{\text{st}})}{W_{O_2}} Q_{H_2}^{-1}
\]  

(21)

Using Eq. (16), Eq. (21) can be written as

\[
\Delta T_{\text{ref}} = \frac{Q_F}{c_p W_{N_2}} = (T_{\text{st}} - T_u) \left( \frac{W_F}{Y_{F,1} W_{N_2} \xi_{\text{st}}} \right) \left( 1 + 4r \frac{Q_{H_2}}{Q_F} \right)^{-1} = \frac{T_{\text{st}} - T_u}{X_{F,1} \xi_{\text{st}}} \left( 1 + 4r \frac{Q_{H_2}}{Q_F} \right)^{-1}
\]  

(22)

Thus \( G_F = 1 \), and \( T_{\text{st}} \) given by Eq. (19) is rewritten as

\[
\tau_{\text{st}} = X_{F,1} \xi_{\text{st}} \left( 1 + 4r \frac{Q_{H_2}}{Q_F} \right)
\]  

(23)

The influence of hydrogen on critical conditions of extinction are obtained at fixed values of \( T_{\text{st}} \) and \( \xi_{\text{st}} \) and at various values of \( r \). The calculations are done with either \( X_{H_2,2} = 0 \) or \( X_{H_2,1} = 0 \). Equation (5) is used to obtain \( x_{\text{st}} \) and \( \xi_{H_2,\text{st}} \). At a selected value of \( r \), Eqs. (20) and (41) are used to obtain \( X_{F,1} \) and \( X_{O_2,2} \). Equation (15) is then used to calculate either \( X_{H_2,2} \) or \( X_{H_2,1} \).
In the asymptotic limit of large values of Damköhler numbers characterizing the rates of chemical reactions, the flame structure is presumed to be made up of, an “inert” convective-diffusive outer structure of thickness of the order of unity, and a thin diffusive reactive inner structure where chemical reactions take place. Asymptotic analysis of these layers gives the critical conditions of extinction. A characteristic diffusion time \( \chi^{-1} \) deduced from the spatial gradient \( \xi \) can be written as:

\[
\chi = 2\left[\frac{\lambda}{\rho c_p}\right]|\nabla \xi|^2,
\]

where \( \lambda \) is the thermal conductivity, and \( \rho \) the density. The quantity \( \chi \) also represents the scalar dissipation rate and plays a central role in asymptotic analyses [2, 3, 12].

4 Outer Structure

In the asymptotic analysis the concentrations of the reactive species CH\(_4\), O\(_2\), H\(_2\) CO\(_2\), and H\(_2\)O are presumed to be of the order of unity. The profiles of concentration of these five reactive species represent the outer structure of the flame. Figure 3 shows the outer structure in the limit of large values of the Damköhler number with oxidizer leakage from the reaction zone to the leading order. Let the flame sheet be located at \( x = x_p \) and \( \xi = \xi_p \). At \( x_p \) there is complete consumption of CH\(_4\), H\(_2\), but O\(_2\) leaks through the reaction zone to the leading order. At \( x_p \), \( X_{O_2} = X_{O_2,p} \). At \( x_{p+} \),

![Figure 3: Schematic of the outer structure with oxidizer leakage to the leading order.](image)

and H\(_2\), but O\(_2\) leaks through the reaction zone to the leading order. At \( x_p \), \( X_{O_2} = X_{O_2,p} \). At \( x_{p+} \),
the gradients are

\[
\begin{align*}
\frac{dX_F}{dx} &= \frac{dX_F}{d\xi} \frac{d\xi}{dx} = \frac{X_{F,1}}{1 - \xi_p} \frac{d\xi}{dx} \\
\frac{dX_O}{dx} &= \frac{dX_O}{d\xi} \frac{d\xi}{dx} = \frac{X_{O,2}}{1 - \xi_p} \frac{d\xi}{dx} \\
\frac{1}{Le_{H_2}} \frac{dX_{H_2}}{dx} &= \frac{1}{Le_{H_2}} \frac{dX_{H_2}}{d\xi} \frac{d\xi}{dx} = \frac{1}{Le_{H_2}} \frac{X_{H_2,1}}{1 - \xi_{H_2,p}} \frac{d\xi}{dx} \\
\frac{dX_{CO}}{dx} &= \frac{dX_{CO}}{d\xi} \frac{d\xi}{dx} = -\frac{X_{CO,2}}{1 - \xi_p} \frac{d\xi}{dx} \\
\frac{dX_{H_2O}}{dx} &= \frac{dX_{H_2O}}{d\xi} \frac{d\xi}{dx} = -\frac{X_{H_2O,1}}{1 - \xi_p} \frac{d\xi}{dx} \\
\frac{d\tau}{dx} &= \frac{d\tau}{d\xi} \frac{d\xi}{dx} = -\frac{\tau_p}{1 - \xi_p} \frac{d\xi}{dx}
\end{align*}
\]  \tag{25}

At \(x_p\) the gradients are

\[
\begin{align*}
\frac{dX_F}{dx} &= 0 \\
\frac{dX_O}{dx} &= \frac{dX_O}{d\xi} \frac{d\xi}{dx} = -\frac{(X_{O,2} - X_{O,2,p})}{\xi_p} \frac{d\xi}{dx} \\
\frac{1}{Le_{H_2}} \frac{dX_{H_2}}{dx} &= \frac{1}{Le_{H_2}} \frac{dX_{H_2}}{d\xi} \frac{d\xi}{dx} = \frac{1}{Le_{H_2}} \frac{X_{H_2,2}}{\xi_{H_2,p}} \frac{d\xi}{dx} \\
\frac{dX_{CO}}{dx} &= \frac{dX_{CO}}{d\xi} \frac{d\xi}{dx} = -\frac{X_{CO,2}}{\xi_p} \frac{d\xi}{dx} \\
\frac{dX_{H_2O}}{dx} &= \frac{dX_{H_2O}}{d\xi} \frac{d\xi}{dx} = -\frac{X_{H_2O,2}}{\xi_{H_2O,2}} \frac{d\xi}{dx} \\
\frac{d\tau}{dx} &= \frac{d\tau}{d\xi} \frac{d\xi}{dx} = \frac{\tau_p}{\xi_p} \frac{d\xi}{dx}
\end{align*}
\]  \tag{26}

Balance equation for carbon across the reaction zone at \(x = x_p\) is

\[
\left( \frac{dX_F}{dx} + \frac{dX_{CO}}{dx} \right) = 0
\]  \tag{27}

This gives

\[
X_{CO,2,p} = X_{F,1,1} \xi_p
\]  \tag{28}

Balance equation for oxygen across the reaction zone at \(x = x_p\) is

\[
\left( 2 \frac{dX_O}{dx} + 2 \frac{dX_{CO}}{dx} + \frac{dX_{H_2O}}{dx} \right) = \left( 2 \frac{dX_O}{dx} + 2 \frac{dX_{CO}}{dx} + \frac{dX_{H_2O}}{dx} \right)
\]  \tag{29}

This gives

\[
X_{H_2O,p} = 2X_{O,2,1} (1 - \xi_p) - 2X_{O,2,p} - 2X_{F,1,1} \xi_p
\]  \tag{30}

Balance equation for hydrogen across the reaction zone at \(x = x_p\) is

\[
\left( 2 \frac{dX_F}{dx} + \frac{1}{Le_{H_2}} \frac{dX_{H_2}}{dx} + \frac{dX_{H_2O}}{dx} \right) = \left( 2 \frac{dX_F}{dx} + \frac{1}{Le_{H_2}} \frac{dX_{H_2}}{dx} + \frac{dX_{H_2O}}{dx} \right)
\]  \tag{31}
This gives

\[
[X_{H_2,2} + \xi_{H_2,p} (X_{H_2,1} - X_{H_2,2})] \frac{\xi_p (1 - \xi_p)}{\xi_{H2,p} (1 - \xi_{H2,p})} \frac{1}{\sqrt{L\xi_{H2}}} \left\{ \exp \left[ \frac{x_p^2 (1 - L\xi_{H2})}{2} \right] \right\} = 2X_{O_2,2} (1 - \xi_p) - 2X_{O_2,p} - 4X_{F,1}\xi_p. \tag{32}
\]

For a given value of \(\xi_p\), Eqs. (5) is used to evaluate \(x_p\) and \(\xi_{H2,p}\). These values are the used to evaluate \(X_{O_2,p}\), \(X_{CO_2}\) and \(X_{H2O,p}\) using Eqs. (32), (28) and (30). Coupling relations for temperature gives:

\[
\left[ \frac{dT}{dx} + 2G_{H2} \frac{dX_{O_2}}{dx} + (G_F - 4G_{H2}) \frac{dX_F}{dx} \right]_+ = \left[ \frac{dT}{dx} + 2G_{H2} \frac{dX_{O_2}}{dx} + (G_F - 4G_{H2}) \frac{dX_F}{dx} \right]_-. \tag{33}
\]

This gives

\[
\tau_p = (G_F - 4G_{H2}) X_{F,1}\xi_p + 2G_{H2} [X_{O_2,2} (1 - \xi_p) - X_{O_2,p}] \tag{34}
\]

The flame temperature \(T_p\) is

\[
T_p = T_u + \frac{Y_{F,1} (Q_F - 4Q_{H2}) \xi_p}{c_p W_F} + \frac{2Q_{H2} [Y_{O_2,2} (1 - \xi_p) - Y_{O_2,p}]}{c_p W_{O_2}} \tag{35}
\]

where \(Y_{O_2,p} = X_{O_2,p} W_{O_2}/W_{N_2}\). The following are the gradients of various species at \(\xi_p+\)

\[
\frac{dX_F}{d\xi} = \frac{X_{F,1}}{1 - \xi_p} = g
\]

\[
\frac{dX_{O_2}}{d\xi} = -\frac{X_{O_2,p}}{1 - \xi_p}
\]

\[
\frac{1}{L\xi_{H2}} \frac{dX_{H_2}}{d\xi} = \frac{X_{H_2,1}}{1 - \xi_{H2,p}} \frac{1}{\sqrt{L\xi_{H2}}} \left\{ \exp \left[ \frac{x_p^2 (1 - L\xi_{H2})}{2} \right] \right\} = m 
\]

\[
\frac{dX_{CO_2}}{d\xi} = -\frac{X_{CO_2,p}}{1 - \xi_p}
\]

\[
\frac{dX_{H_2O}}{d\xi} = -\frac{X_{H_2O,p}}{1 - \xi_p}
\]

\[
\frac{d\tau}{d\xi} = -\frac{\tau_p}{1 - \xi_p} = -c
\]

The following are the gradients of various species at \(\xi_p-\)

\[
\frac{dX_F}{d\xi} = 0
\]

\[
\frac{dX_{O_2}}{d\xi} = -\frac{(X_{O_2,2} - X_{O_2,p})}{\xi_p} = -2b
\]

\[
\frac{1}{L\xi_{H2}} \frac{dX_{H_2}}{d\xi} = -\frac{X_{H_2,2}}{\xi_{H2,p}} \frac{1}{\sqrt{L\xi_{H2}}} \left\{ \exp \left[ \frac{x_p^2 (1 - L\xi_{H2})}{2} \right] \right\} = -n
\]

\[
\frac{dX_{CO_2}}{d\xi} = \frac{X_{CO_2,p}}{\xi_p}
\]

\[
\frac{dX_{H_2O}}{d\xi} = \frac{X_{H_2O,p}}{\xi_p}
\]

\[
\frac{d\tau}{d\xi} = \frac{\tau_p}{\xi_p} = d
\]
The Structure of the Reaction Zone

The structure of the reaction zone around $\xi_p$ is described by the four-step reduced chemical-kinetic mechanism. For convenience, the definitions

$$x_i \equiv \frac{X_i}{L e_i}, \quad \omega_k \equiv \frac{W_{N_k}^2 w_k}{\rho^2}, \quad Q_k \equiv \frac{(-\Delta H_k)}{Q_F}$$

are introduced, where $(-\Delta H_k)$ is the heat release in the overall step $k$ of the reduced mechanism. From the definition of $Q_k$ it follows that $Q_I + Q_{II} + Q_{III} + 2Q_{IV} = 1$. The scalar dissipation rate, defined by Eq. (24), is evaluated at $\xi_p$ and represented by $\chi_p$. Changes in the values of the scalar dissipation rate in the reaction zone are neglected [9]. The differential balance equations for the fuel, H-radicals, CO, and $H_2$ written in terms of the variables defined in Eq. (38) are

$$\left(\frac{\chi_p}{2}\right) \left(\frac{W_N}{\rho}\right) \frac{d^2x_F}{d\xi^2} = \omega_I,$$

$$\left(\frac{\chi_p}{2}\right) \left(\frac{W_N}{\rho}\right) \frac{d^2x_{II}}{d\xi^2} = 2\omega_I + 2\omega_{III} - 2\omega_{IV},$$

$$\left(\frac{\chi_p}{2}\right) \left(\frac{W_N}{\rho}\right) \frac{d^2x_{CO}}{d\xi^2} = -\omega_I + \omega_{II},$$

$$\left(\frac{\chi_p}{2}\right) \left(\frac{W_N}{\rho}\right) \frac{d^2x_{H_2}}{d\xi^2} = -4\omega_I - \omega_{II} - \omega_{III} + 3\omega_{IV}$$

Coupling relations satisfying the conservation of chemical elements and energy can be written as

$$d^2(4x_F + x_{H}/2 + x_{CO} + x_{H_2} - 2x_{O_2})/d\xi^2 = 0,$$

$$d^2(x_{CO_2} + x_F + x_{CO})/d\xi^2 = 0,$$

$$d^2(x_{H_2O} + 2x_F + x_{H}/2 + x_{H_2})/d\xi^2 = 0,$$

$$d^2(\tau + x_F + q_H x_{H} + q_{CO} x_{CO} + q_{H_2} x_{H_2})/d\xi^2 = 0,$$

where $q_H = (3Q_{III} + Q_{IV})/4$, $q_{CO} = (2Q_{II} + Q_{III} + Q_{IV})/2$, and $q_{H_2} = (Q_{III} + Q_{IV})/2$.

The global reactions of the reduced four-step mechanism take place in various layers within the reaction zone. Figures 4 and 5 are schematic illustrations of the flame structure for hydrogen added to the fuel side and air side respectively. The forward and backward steps of the elementary reaction 1 take place everywhere in the reaction zone. The ratio $w_{1b}/w_{1f} = k_{1b}X_{OH}X_O/(k_{1f}X_{H}X_{O_2})$ is presumed to be of the order of unity. Using Eqs. (2) and (7) it can be deduced that this ratio is equal to $X_H^2/R^2$, where $R$ is given by the expression

$$R = \frac{(K_1 K_2 K_3)^{1/2} X_{H_2}^{3/2} X_{O_2}^{1/2}}{X_{H_2O}}.$$

The value of $X_H$ is of the order of $R$ everywhere in the reaction zone. The quantity $R$ is presumed to be of the order of $\delta$. Asymptotic analysis of the inner layer and the oxidation layer provide sufficient conditions for calculating the scalar dissipation rate at extinction.
Figure 4: Schematic illustration of the structure of the reaction zone around $\xi_p$ for hydrogen added to the fuel stream. The figure shows the inner layer and the oxidation layer.

Figure 5: Schematic illustration of the structure of the reaction zone around $\xi_p$ for hydrogen added to the oxidizer stream. The figure shows the inner layer and the oxidation layer.
6 Asymptotic Analysis of the Oxidation Layer

In the oxidation layer the concentration of fuel is negligibly small, therefore the influence of the global step I on the structure of this layer is neglected. The concentration of H-radicals is presumed to maintain steady-state. As a consequence of introducing steady-state approximation for H, Eq. (39) gives

$$X_H = R(1 - k_5C_M/k_{11})^{1/2}$$  \hspace{1cm} (42)

where $R$ is defined in Eq. (41). These approximations reduce the four-step mechanism to a two-step mechanism, which can be written as

$$\text{CO} + \text{H}_2 \text{O} \rightleftharpoons \text{CO}_2 + \text{H}_2, \quad \text{II}$$
$$\text{O}_2 + 2 \text{H}_2 \rightleftharpoons 2 \text{H}_2\text{O}. \quad \text{III}'$$

The reaction rate of the global step III’ is the same as that for the global step III of the four-step mechanism. The differential equations describing the structure of this layer are

$$\frac{d^2x_{\text{CO}}}{d\xi^2} = \frac{D_{\text{III}}SG_{\text{III}}x_{\text{H}_2}^{1/2}(x_{\text{CO}} - \alpha G_ax_{\text{H}_2}) (1 - G_b\kappa)^{1/2}}{\sqrt{2g}},$$
$$\frac{d^2(x_{\text{CO}} + x_{\text{H}_2})}{d\xi^2} = \frac{D_{\text{III}}G_{\text{III}}x_{\text{H}_2}^{3/2} (1 - G_b\kappa)^{1/2}}{\sqrt{2g}},$$  \hspace{1cm} (43)

The equation for hydrogen is

$$\frac{d^2x_{\text{H}_2}}{d\xi^2} = \frac{D_{\text{III}}x_{\text{H}_2}^{1/2} (1 - G_b\kappa)^{1/2} [G_{\text{III}}x_{\text{H}_2} - SG_{\text{III}}(x_{\text{CO}} - \alpha G_ax_{\text{H}_2})]}{\sqrt{2g}}$$  \hspace{1cm} (44)

Here $g$ is defined in Eq. (36). The quantities $D_{\text{III}}, S, \alpha,$ and $\kappa$ are defined as

$$D_{\text{III}} \equiv \frac{2^{5/2} \rho^{g/2} k_0^0 C_M^0 [K_1^0 K_2^0 K_3^0 K_4^0]^{1/2} \rho^{2/3} \rho^{1/2} L_{\text{H}_2} x_{\text{CO}_2,\text{p}}^{3/2} X_{\text{H}_2\text{O}_2,\text{p}}^{3/2}}{\chi_p X_{\text{H}_2\text{O}_2,\text{p}} W_{\text{N}_2}},$$
$$S \equiv \frac{2^{1/3} \rho^{1/2} k_0^0 C_M^0 K_3^3 L_{\text{H}_2} X_{\text{CO}_2,\text{p}}}{\rho^{1/2} k_0^0 C_M^0 K_3^3 L_{\text{H}_2} X_{\text{CO}_2,\text{p}}},$$
$$\alpha \equiv \frac{K_3^3 X_{\text{CO}_2,\text{p}} L_{\text{H}_2}}{K_0^0 X_{\text{H}_2\text{O}_2,\text{p}} L_{\text{CO}}},$$
$$\kappa \equiv \frac{K_0^0 C_M^0}{K_1^1}.$$  \hspace{1cm} (45)

Here $k_0^0$ and $K_0^0$ are, respectively, the reaction rate coefficients and equilibrium constants of the elementary reaction evaluated at $T = T_p$. The quantities $G_{\text{II}}, G_{\text{III}}, G_a,$ and $G_b$ are defined as

$$G_{\text{II}} = \frac{\rho k_0^0 K_1^{1/2} K_2^{1/2}}{\rho^0 k_0^0 K_1^{0/2} K_2^{0/2}},$$
$$G_{\text{III}} = \frac{\rho k_0^0 K_1^{1/2} K_2^{1/2} K_3^{1/2}}{\rho^0 k_0^0 K_1^{0/2} K_2^{0/2} K_3^{0/2}},$$
$$G_a = \frac{K_3 K_0}{K_1^2 K_6},$$
$$G_b = \frac{k_0^0 k_3 C_M}{k_1^1 k_3^2 C_M}.$$  \hspace{1cm} (46)
The approximation $T^{\alpha_n} = T_m^{\alpha_n} \exp (\alpha_n \exp (-\alpha_n T_m / T))$ is introduced. Here $T_m$ is a reference temperature.

$$G_{II} = \exp \left[ -m_{II} \left( 1/T - 1/T_p \right) \right]$$
$$G_{III} = \exp \left[ -m_{III} \left( 1/T - 1/T_p \right) \right]$$
$$G_a = \exp \left[ -m_a \left( 1/T - 1/T_p \right) \right]$$
$$G_b = \exp \left[ -m_b \left( 1/T - 1/T_p \right) \right]$$

The quantities $m_{II}$, $m_{III}$, $m_a$, and $m_b$ are defined as

$$m_{II} = \frac{(E_{1f} - E_{1b} + E_{2f} - E_{2b})/2 + E_{6f}}{R^0} + (\alpha_6 - 1) T_m$$
$$m_{III} = \frac{(E_{1p} - E_{1b} + E_{2f} - E_{2b})/2 + E_{3f} - E_{3b}}{R^0} + (\alpha_5 - 2) T_m$$
$$m_a = \frac{E_{2f} - E_{3b} - E_{6f} + E_{6b}}{R^0}$$
$$m_b = -\frac{E_{1b}}{R^0} + (\alpha_5 - 1) T_m$$

Here $R^0 = 8.314 \text{ J/(mol·K)}$ is the universal gas constant. It follows from Table 1, with $T_m = 1600 \text{ K}$

$$m_{II} = 4925 \text{ K}, \quad m_{III} = -7519 \text{ K}, \quad m_a = 3632 \text{ K}, \quad m_b = -11335 \text{ K}.$$  (49)

The coupling relations deduced from Eq. (40) are

$$d^2(x_{CO} + x_{H_2} - 2x_{O_2})/d\xi^2 = 0,$$
$$d^2(x_{CO} + x_{CO})/d\xi^2 = 0,$$
$$d^2(x_{H_2O} + x_{H_2})/d\xi^2 = 0,$$
$$d^2[\tau + q(x_{CO} + x_{H_2})]/d\xi^2 = 0.$$  (50)

where $q_{CO}$ and $q_{H_2}$ in Eq. (40) are replaced by $q = (Q_{II} + Q_{III} + Q_{IV})/2$ in Eq. (50) because their numerical values are nearly the same. The value of $q$ is 0.33.

The expansions

$$\xi - \xi_p = \epsilon \eta, \quad x_{CO} = \epsilon 2g z_{CO},$$
$$x_{H_2} = \epsilon 2g [z_{H_2} - n\eta/(2g)]$$

are introduced, where $\epsilon$ is small and the variables $\eta$, $z_{CO}$, and $z_{H_2}$ are presumed to be of the order of unity. Matching the profiles of CO and H$_2$ in the oxidation layer with those in the outer structure in the region $\xi < \xi_p$ gives

$$dz_{H_2}/d\eta = dz_{CO}/d\eta = 0 \quad \text{as} \quad \eta \to -\infty.$$  (52)

The expansions for $x_{O_2}$ and $\tau$ are written as

$$x_{O_2} = X_{O_2,p}/Le_{O_2} + \epsilon (g z_{CO} + g z_{H_2} - 2b\eta),$$
$$\tau = \tau_p - \epsilon [2gq (z_{CO} + z_{H_2}) - d\eta].$$  (53)

The expansions shown in Eq. (53) are so chosen that they satisfy the coupling relations shown in Eq. (50) and match with the slopes shown in Eq. (37) as $\eta \to -\infty$. 

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The small expansion parameter $\epsilon$ is so chosen that it can be calculated from the equation

$$\epsilon = D_{III}^{-2/5}. \quad (54)$$

Introducing the expansions given by Eq. (51) into Eqs. (43) and (44) gives

$$\frac{d^2 z_{CO}}{d\eta^2} = S G_{II} (1 - G_b \kappa)^{1/2} \left( z_{H_2} - \frac{nn}{2g} \right)^{1/2} \left[ z_{CO} - \alpha G_a \left( z_{H_2} - \frac{nn}{2g} \right) \right],$$

$$\frac{d^2 z_{H_2}}{d\eta^2} = (1 - G_b \kappa)^{1/2} \left( z_{H_2} - \frac{nn}{2g} \right)^{1/2} \left\{ G_{III} \left( z_{H_2} - \frac{nn}{2g} \right) - S G_{II} \left[ z_{CO} - \alpha G_a \left( z_{H_2} - \frac{nn}{2g} \right) \right] \right\}. \quad (55)$$

In view of Eqs. (7) and (53)

$$T = T_p - \epsilon [2gq (z_{CO} + z_{H_2}) - d\eta] \Delta T_{ref}, \quad (56)$$

where $\Delta T_{ref}$ is given by Eq. (22). At $\eta = 0$, the values of $T$, $z_{CO}$ and $z_{H_2}$ are $T^0$, $z_{CO}^0$ and $z_{H_2}^0$ respectively. It follows from Eq. (56) that

$$T^0 = T_p - 2\epsilon gq \left( z_{CO}^0 + z_{H_2}^0 \right) \Delta T_{ref}, \quad (57)$$

It follows from Eq. (56) that the leading order expansion for

$$1/T - 1/T_p = \epsilon [2gq (z_{CO} + z_{H_2}) - d\eta] \Delta T_{ref} / T_p^2, \quad (58)$$

The values of $G_{II}, G_{III}, G_a,$ and $G_b$ is evaluated using using Eqs. (47), (49), and (58). Equation (42) shows that the steady-state concentration of H is proportional to the square root of $1 - k_5 C_M / k_{1f}$. The value of H is zero when $k_5 C_M / k_{1f} = 1$. The temperature at which this is achieved is represented by $T_c$. Thus the steady-state approximation for H breaks down at $\eta = \eta_c$ where $z_{CO} = z_{CO,c}$ and $z_{H_2} = z_{H_2,c}$. Thus

$$2gq \left( z_{CO,c} + z_{H_2,c} \right) - d\eta_c = \frac{T_p^2 \left( -\ln \kappa \right)}{\epsilon \Delta T_{ref} \left( -m_b \right)}. \quad (59)$$

Equations (55) should satisfy the boundary condition given by Eq. (52) at $\eta = \eta_c$.

A set of boundary conditions at $\eta = 0$ for Eq. (55) can be obtained by considering the jump conditions across the inner layer. In the inner layer if the influence of the global steps II and III on the structure of this layer is neglected, then from Eq. (39) the coupling relations $d^2(x_F + x_{CO})/d\xi^2 = 0$ and $d^2(x_F + 3x_{H_2}/2 + x_{H_2})/d\xi^2 = 0$ are obtained. Integrating this equation once and matching the slopes with those given by Eq. (36) gives the result $d(x_F + x_{CO})/d\xi = g$, and $d(x_F + 3x_{H_2}/2 + x_{H_2})/d\xi = g + m$. Using the expansions given by Eq. (51), the boundary conditions

$$dz_{CO}/d\eta = 1/2,$$

$$dz_{H_2}/d\eta = 1/2 + n/(2g) + m/(2g)$$

are obtained at $\eta = 0$.  

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Equation (55) are integrated numerically employing the boundary conditions given by Eqs. (52) and (60). The results are used to evaluate the values of \( z_{CO}^0 \) and \( z_{H_2}^0 \).

The analysis of the inner layer is identical to that of Bai and Seshadri [10]. The procedure for obtaining the scalar dissipation rate at extinction is identical to that in Ref. [10]. The results are described in the next section.

7 Results

Figure 6 shows the classical C-Shaped curve where \( T_p \) and \( T^0 \) are plotted as functions of \( \chi_p^{-1} \), for \( Y_{H_2,1} = 0 \) and for various values of \( Y_{H_2,2} \). For a given value of \( \chi_p^{-1} \) two solutions are obtained for \( T_p \) and \( T^0 \). Critical conditions of extinction are given by the value of \( \chi_p^{-1} \) where these solutions merge. Figures 7 and 8 show critical conditions for extinction. In these figures the extinction strain rate, \( a_{q,0} \) normalized by the extinction strain rate without hydrogen addition, \( a_{q,0} \) is plotted as a function of \( r \). In these figures the symbols represent experimental data, and the curves represent predictions obtained using the asymptotic model. These curves are boundaries separating flammable regions below from nonflammable regions above. Figure 7 shows critical conditions of extinction for hydrogen added to the oxidizer side and Fig. 8 shows results for hydrogen added to the fuel stream. The predictions in Fig. 7 agree well with experimental data, while the predictions shown in Fig. 8 do not agree with experiments.
Figure 7: The normalized strain rate at extinction, $a_q/a_{q,0}$ as a function of $r$ at fixed $\xi_{st} = 0.055$, and $T_{st} = 2000\text{K}$. Hydrogen is added to the oxidizer stream but not to the fuel stream. The symbols represent experimental data and the curve represents predictions of the asymptotic model.
Figure 8: The normalized strain rate at extinction, \( \frac{a_q}{a_{q,0}} \) as a function of \( r \) at fixed \( \xi_{\text{st}} = 0.055 \), and \( T_{\text{st}} = 2000 \text{K} \). Hydrogen is added to the fuel stream. The symbols represent experimental data and the curves represent predictions of the asymptotic model.
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