Boundary Layer Effect on the Correlation of Spread Rate Data in Opposed Flow Flame Spread

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In opposed-flow flame spread over solid fuels, an indefinite increase in flow velocity eventually leads to flame extinguishment. While the chemical time is independent of the flow velocity, the residence time of the oxidizer at the flame leading edge is inversely proportional to the flow velocity, and, therefore, a competition between the two leads to a situation where finite-rate kinetics dominates the flame spread behavior, leading to blow-off extinguishment. The ratio of the two competing times (residence time to chemical time), known as the Damkohler number, captures this finite-rate effect and has been used to correlate the non-dimensional spread rate with opposing flow velocity and ambient oxygen level. Although these correlations explain the behavior observed in the experiments there is considerable spread in the correlations found in literature despite the use of several variations of the definition of the Damkohler number. With all the progress made in this area, it is still not possible to predict the blow-off extinction velocity for a given fuel at any given oxidizer condition.

In this work we present new flame spread data over ashless filter paper acquired in a eight meter tall vertical steel chamber in which the sample is moved at a command velocity to create a desired opposing flow. The developing boundary layer over the fuel sample and the relative humidity in the chamber are shown to have a significant effect on the measured spread rate and it does not correlate at all with opposing flow. Once the data is adjusted for humidity, and an effective flow velocity that incorporates the developing boundary layer is substituted for opposing velocity, the correlation is shown to improve drastically. Given the importance of the boundary layer development, much of the data in literature that do not mention the development length must be cautiously used.

1. Introduction

Flame spread over solid fuels in an opposed-flow environment has been investigated for over four decades [1],[2],[3] for understanding the fundamental nature of hazardous fire spread. The appeal for this configuration stems from the fact that flame spread rate remains steady even if the flame itself may grow in size. The configuration can be further simplified by restricting fuel thickness to the thermally thin limit whereby the entire thickness of the fuel can be assumed to be uniformly heated by the spreading flame.

The simplified problem still retains the richness of exhibiting characteristics of a full range of flame spread regimes. In the thermal regime, the spread rate is characterized by a balance between conduction from the gas phase and the energy required to heat up the virgin fuel from the ambient to the vaporization temperature. The well-known de Ris-Delichatsios formula [4][5] expresses the flame spread rate in terms of fuel thickness, thermodynamic properties, and a non-dimensional temperature coefficient that depends on the flame temperature and the vaporization temperature. A very interesting feature of the thermal regime is that the flame spread rate is independent of the opposing flow velocity.

If the opposing flow velocity is reduced, buoyancy creates a permanent opposing flow for a downward spreading flame. However, in the microgravity environment, the opposing flow can be very small or even completely absent. It has been shown that in addition to gas phase conduction, the radiative effects become important [6],[7],[8] and flame spread rate actually depends on the flow velocity. A closed-form formula for the
flame spread rate has been proposed by Bhattacharjee et. al [9] in this radiative regime that predicts flame extinguishment for fuels exceeding certain critical thickness.

An implicit assumption in the thermal regime is that the flow velocity is low enough to allow sufficient time for combustion to be complete. That is, combustion and pyrolysis can be assumed to be infinitely fast compared to other processes. However, when the opposed flow is indefinitely increased, the residence time at the flame leading edge can become too small for this assumption to hold and the effect of finite-rate kinetics can become significant. Flame spread rate begins to decrease in this kinetic regime and the flame eventually extinguishes at a sufficiently high velocity. Use of phenomenological arguments led Frey and Tien[10], Altenkirch et. al[11], Fernandez-Pello et. al[12] and Wichman[13] to correlate the spread rate, non-dimensionalized using its thermal limit, with the Damkohler number, the ratio of the residence time and chemical time at the flame leading edge.

Although successful in correlating a given set of data, such correlations are far from universal due to several complicating reasons: First, the flame temperature appears both in the spread rate expression in the thermal regime and in defining a chemical time, the denominator of the Damkohler number. While de Ris solution[4] requires the use of a linearized flame temperature (which is considerably higher than the adiabatic flame temperature for complete combustion), adiabatic flame temperature or even the equilibrium flame temperature have been used in the expression for chemical time. Second, for flame spread in a quiescent environment in a normal or partial gravity environment, the opposing flow velocity is not known and must be scaled by balancing the inertia term with buoyancy in the momentum equation [14], [11]. In a forced convective situation, the free stream velocity may be known, but the velocity gradient encountered by the flame will depend on the boundary layer development upstream of the flame. For flame spread over thick fuels in the thermal regime, Wichman[15] explored the effect of the velocity gradient at the flame leading edge while West et. al [16] proposed the concept of an effective velocity seen by the flame based on the velocity profile encountered by the flame.

For thin fuels, spread rate being independent of flow velocity in the thermal limit, the developing boundary layer needs to be incorporated only in the Damkohler number. Etoh et. al [17] used an unsteady numerical simulation to show that the flame spread can be considered pseudo-steady despite the fact that the flame encounters a developing boundary layer with a varying effective flow velocity. They defined an effective velocity as the local velocity where the peak flame temperature occurs and showed that the spread rate for different development lengths and flow velocity correlate well with the numerically obtained effective velocity. Bhattacharjee et al used a scaling approach to define an effective velocity as the velocity seen by the flame one length scale above the fuel surface.

![Image](image_url)
Using data from a steady-state numerical model, they proposed a formula for this velocity in terms of the development length of the boundary layer development length, free-stream velocity, and fluid properties.

In this work we present newly acquired data from the flame tower at SDSU. The data provides strong indication that the boundary layer development length must be considered in correlating spread rate with opposed-flow velocity.

2. Effective Velocity

In order to obtain an expression for the effective velocity seen by the flame, consider the leading edge of the spreading flame as shown in Fig 1. At a distance \( x \) from the leading edge of the fuel sample, the flame is shown to be embedded inside the boundary layer of thickness \( \delta \). Based on numerical evidence [16][17] that the diffusion length scale \( L_g \) at the flame leading edge plays a critical role in defining flow experienced by the flame, we define velocity at a distance \( L_g \) from the fuel surface at the leading edge as the effective velocity (see Fig. 6).

Assuming a linear velocity profile, we can express \( V_{eff} \) in terms of \( L_g \) and \( \delta \) as:

\[
V_{eff} \approx \frac{L_g}{\delta} V_g
\]  
(1)

A balance between the forward conduction and the advection term of the energy equation establishes the diffusion length scale

\[
\frac{\partial}{\partial x} \left( \rho u c_p T \right) - \frac{\partial}{\partial x} \left( \lambda_g \frac{\partial T}{\partial x} \right) \Rightarrow \frac{\rho V_g c_p \Delta T}{L_g} \sim \frac{\lambda_g \Delta T}{L_g} \Rightarrow L_g \sim \frac{\alpha_g}{V_g} \]  
(2)

The boundary layer thickness can be scaled by balancing the inertial term with the friction term of the momentum equation.

\[
\frac{\partial}{\partial x} (\rho u^2) - \frac{\partial}{\partial y} (\mu_g \frac{\partial u}{\partial y}) \Rightarrow \frac{\rho V_g^2}{x_d} \sim \frac{\mu_g V_g}{\delta^2} \Rightarrow \delta \sim \frac{x_d}{\sqrt{Re_x}} \]  
(3)

Substituting expressions (2) and (3) in Eq (1) and introducing Prandtl number \( Pr = \nu / \alpha \), we obtain
To improve the correlation, we notice that the diffusion length scale is also affected by the developing boundary layer. If the correct velocity scale is $V_{\text{eff}}$, it should be used in Eq. (2) as well in place of $V_g$. That is,

$$L_{g,\text{eff}} \sim \frac{\alpha_g}{V_{\text{eff}}}$$  \hspace{1cm} (5)

Substituting this expression in Eq. (1) and a little manipulation leads to a different power law.

$$V_{\text{eff}} \sim \frac{V_g}{\text{Pr Re}^{1/4}}$$  \hspace{1cm} (6)

Comparison with several sets of numerical data led [18] to an empirical expression using a one-third power,
This final expression, Eq. (7), seems to work satisfactorily as can be seen from the collapse of the numerical spread rate data in Fig. 2 for flame spread over PMMA sheets in a 50% oxygen and 50% nitrogen environment at 100 kPa for different development lengths, which can be seen to play a very significant role in determine the blow-off velocity. When the development length is 30 mm as opposed to 60 mm, the spread rate is consistently depressed as the effective velocity is much higher. Except for the very last near-limit computation, which carries more numerical uncertainties, all other data point approximately align around one universal line.

3. The SDSU Flame Tower
Most opposed-flow flame experiments have been performed in wind tunnels. However, creating a low velocity field with a known profile is a challenging task. At SDSU we built a 8 m tall vertical steel chamber with a 45 cm x 45 cm square cross-section, which we call the flame tower [19] (see Fig. 4), inside which a fuel sample mounted on a cart can be traversed up or down with a prescribed velocity. The cart carrying the experimental package is connected by a string going over a pulley at the top of the tower to a counter weight that moves up and down through a vertical tube in opposite direction of the cart. A stepper motor (step angle 0.028125 degree, 12800 steps per shaft revolution, 120 VAC, 5.0 A, max holding torque 5.4 N-m) housed at the top of the tower creates the desired motion by winding or unwinding a separate string connected to the cart. The power supply and the connection to the serial port of the indexer of the motor is run through an electrical seal.

The velocity of the cart was measured by analyzing a digital high-speed video of a measuring tape attached to the rail from a camera mounted on the cart. The acceleration, velocity and deceleration profiles matched the command profiles almost exactly. A hot wire anemometer was connected to the cart and placed at several
locations to ascertain the flow velocity, which was tested to be reasonably uniform [20] over a 10 cm by 10 cm area upstream of the fuel sample.

The cart, shown in Fig. 4, carries an assembly of fuel sample, igniter, diagnostic system, and an onboard computer all powered by a battery and remotely accessed from outside through a wireless network. The fuel sample, 2 cm wide and 12 cm long, is sandwiched between two thin stainless steel sheets with rectangular cut outs held by an arm attached to the cart. A Kanthal (iron-chromium-aluminum, FeCrAl) alloy igniter wire at the top of the sample holder is connected to the ignition circuit controlled by a micro-controller through the same arm. A web cam captures a high resolution video of the flame spread. Using the NASA Spotlight video analysis software[21], a centerline spot is tracked as the flame passes over a predetermined location (with the desired length of boundary layer development length upstream). The pixel location vs. time data generated by the software is converted into a location vs. time plot and a linear curve fit produces the average spread rate. Typically, a distance of 15 mm is found to be sufficiently long to produce an average spread rate at a given location.

4. Flame Tower Data and Discussion

The data reported in this work is for flame spread over ashless filter paper (Whatman grade 1) of thickness 180 μm with atmospheric air at ambient pressure as the oxidizer. A typical video analysis of the flame spread using the Spotlight imaging software is shown in Fig. 5. As can be seen from the figure, tracking a spot at the centerline over a 10 s of spread (over a distance of 13.6 mm) produces a linear fit with a high degree of correlation.

Downward flame spread in a quiescent environment is characterized by buoyancy generated opposing flow and the spread rate or the flame structure is found to be independent of the location of flame front[22] with respect to the leading edge of the fuel sample. Even if a forced flow is present, buoyancy effect can still be dominant. In the flame tower, when a sample is moved down with a speed below 20 cm/s, there is no discernible change in flame spread rate. The same behavior can also be observed in the data of Fernandez-Pello et al. [12].
To experimentally investigate the effect of the boundary layer development, the relative flow velocity must be sufficiently large to dominate over the buoyancy induced opposing flow. In the flame tower, when the sample is moved at a speed over 25 cm/s, the spread rate starts decreasing leading to blow-off extinction at about 50 cm/s. Based on this observation, we select 30 cm/s and 40 cm/s as the cart velocities to study the effect of the boundary layer development on the spread rate and flame structure.

Top view of the flame for a downward spread experiment in a quiescent environment (zero cart velocity) for three different runs are compared with the corresponding flame images for a cart velocity of 40 cm/s at two different development lengths (2 cm and 11 cm) in Fig. 6. Flame images for pure downward spread look remarkably similar for the cart velocity of 40 cm/s when the development length, $x_d$, is 11 cm. However, the flame size significantly decreases at a lower development length of $x_d = 2$ cm as can be seen from Fig. 6. In fact the flame
size continues to shrink as \( x_d \) decreases until a critical size is reached at about \( x_d = 1 \) cm when the flame extinguishes.

The flame position along the centerline of the sample, tracked with the Spotlight software [21], is shown in Fig. 7 along with the instantaneous spread rate obtained by differentiating the data and smoothing it using a moving average over 4 consecutive data points spanning 4 s of spread. For pure downward spread during the first 35 seconds, the spread rate can be seen to be relatively steady around and average spread rate of 1.92 mm/s.

When the cart starts its downward motion and accelerates to 40 cm/s, the flame undergoes transition adjusting to the relative opposing flow created by the motion of the cart. However, even after the transition period, the spread rate can be seen to continuously decrease. This trend has been verified to be repeatable in multiple runs of the same experiment. It supports the result of the scale analysis where the effective velocity seen by the flame can be seen to continuously increase (see Fig. 2) as the boundary layer development length decreases during the spread. The flame finally extinguishes before the cart reaches the bottom of the tower. As far as we know this is the first experimental evidence of flame extinguishment caused by variation in boundary layer thickness.
To quantitatively demonstrate that the boundary layer development length is indeed the reason behind the drop in flame spread rate in Fig. 7, spread rate measured for two different development lengths at two cart velocities are plotted in Fig. 8 with each data point being the average of 3 different runs. For the larger development length of 11 cm, the standard deviation in the spread rate is less than 0.03 mm/s; however, for the shorter development length of 2 cm, the standard deviation increases to about 0.19 mm/s. Because of the flow constriction caused by the cart, the cart velocity is not the same as the free-stream velocity seen by the leading edge of the sample. Using a velocity probe and Fluent simulation [20], the flow velocity has been shown to be about 1.05 times that of the cart velocity. This constriction factor is incorporated in the data of Fig. 8.

Fig. 8 (a) Spread rate for two different cart velocities when the flame is at two different development lengths. (b) The normalized spread rate correlate well with the effective flow velocity. Fig. 4 The 8 m tall flame tower at SDSU where a fuel sample can be ignited and moved up or down with a desired velocity.
The dimensional plot of Fig. 8(a) shows that depending on the development length $x_d$ the spread rate behaves differently as the opposing flow velocity is increased from 30 cm/s to 40 cm/s. With $x_d = 11 \text{ cm}$ the spread rate goes from 1.66 to 1.67 mm/s. On the other hand, for $x_d = 2 \text{ cm}$ the spread rate goes from 1.50 to 1.31 mm/s. Because of the finite-rate kinetics effect, one would expect the higher opposing flow to cause a decrease in the spread rate. But, ignoring the role played by the development length, what we see is quite the opposite with the spread rate increasing from 1.5 mm/s to 1.67 mm/s as the opposing flow velocity increases from 30 cm/s ($x_d = 2 \text{ cm}$) to 40 cm/s ($x_d = 11 \text{ cm}$). The apparent contradiction can be explained when the effective velocities are calculated as 4.2 cm/s and 2.9 cm/s. That is, even though the opposing flow velocity increases, the effective flow velocity actually decreases in this case. In fact, the apparent lack of correlation between the spread rate and opposing flow velocity in the dimensional plot of Fig. 8(a) disappears when the spread rate, non-dimensionalized, by the pure downward spread rate is plotted against the effective flow velocity as given by Eq. (7).

Given the importance of the role of the developing boundary layer, established in this work, the Damköhler number correlations found in literature where the development length is completely ignored brings into question the validity of such results. Replotting the existing data with the effective velocity in place of the opposing flow velocity is not possible because the development length of the boundary layer when the flame spread rate is measured is not reported in most published work.

5. Conclusion
The effect of a developing boundary layer on spread rate is experimentally established by conducting opposed flow experiments in an eight meter tall closed chamber. The novelty of the experimental set up allows the sample to be moved at a desired speed creating the relative opposed flow velocity. Results from two different development lengths, 2 cm and 11 cm, of the boundary layer and two different opposing flow velocities, 30 cm/s and 40 cm/s, are reported. When the spread rate is plotted against the opposing flow velocity, the result shows non-physical behavior with the spread rate sometime increasing with opposing flow velocity in the kinetic regime where just the opposite is expected. A closed form formula for the effective flow velocity seen by the flame that takes into account the boundary layer development length is used to explain the apparent contradiction. The Damköhler number correlations found in literature for the kinetic regime of opposed-flow flame spread have to be revisited to incorporate this important boundary layer effect before they can be used for any quantitative purpose.

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7. Nomenclature

\( c_g \) Specific heat of gas, kJ/kg\( \cdot \)K

\( c_s \) Specific heat of solid, kJ/kg\( \cdot \)K

\( F \) Flame constant, Eq. (5)

\( f \) Radiation view factor for the gas to the solid preheat region

\( L_g \) Gas-phase diffusion length scale, m

\( T_\infty \) Ambient temperature, K

\( V_g \) Velocity of the oxidizer, m/s

\( V_f \) Absolute spread rate, m/s

\( V_r \) Velocity relative to the flame,

\( \lambda_g \) Gas-phase conductivity evaluated at \( T_v \), kW/m\( \cdot \)K

\( \eta_g \) Non-dimensional flow velocity, Eq. (10)

\( \rho_g \) Gas density evaluated at \( T_v \), kg/m\(^3\)

\( \rho_s \) Solid density, kg/m\(^3\)

\( \tau \) Fuel half-thickness, m

\( \sigma \) Stefan-Boltzman constant, kW/(m\(^2\)\( \cdot \)K\(^4\))

Subscripts

Comb \( \) Combustion

Eff \( \) Effective

Gas \( g \) Gas phase

Rad \( rad \) Radiation

Res \( res \) Residence

Solid \( s \) Solid phase

Th \( th \) Thermal

Vap \( vap \) Vaporization

Greek Symbols

\( \alpha_g \) Thermal diffusivity of gas, evaluated at \( T_v \), m\(^2\)/s

\( \varepsilon \) Surface emissivity

\( V_r = V_{comb}^{eff} + V_f \) Combustion Effective
Works Cited

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