Scaling Turbulent Flame Speeds of Negative Markstein Length Fuel Blends Using Leading Points Concepts

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This work describes analyses of turbulent flame speeds, $S_T$, of negative Markstein length fuel blends using leading point models. In our earlier work we demonstrated that turbulent consumption speed, $S_{T,GC}$ measurements acquired by our group for H$_2$/CO mixtures could be collapsed very well when normalized by the maximum laminar burning velocity of highly stretched flames, $S_{L,max}$. However, systematic differences between $S_{T,GC}/S_{L,max}$ at different pressures remained, and non-quasi-steady behavior in the internal flame structure and mass burning rate at the flame leading points was proposed as a potential mechanism for these differences. The degree of non-quasi-steadiness were quantified with a Damköhler number, defined as the ratio of the chemical time-scale associated with $S_{L,max}$ and a fluid mechanic time-scale. In this paper, we apply these scalings to data sets from the literature which explore pressure and fuel composition effects on $S_T$. Amongst these data sets are new measurements, acquired by our group, of $S_{T,GC}$ for H$_2$/CO mixtures at pressures up to 20 atm. A promising scaling approach for the turbulent flame speed is of the form $$\frac{S_T}{S_{L,max}} = f\left(\frac{u'_\text{rms},r_s}{S_{L,max}},\frac{\tau_{s,\text{max}}}{\tau_{\text{flow}}}\right).$$

However, we also emphasize that it is difficult to differentiate time-scale, length scale, and Reynolds number effects in these correlations (as they lead to similar results) and that additional data, such as with high degrees of preheating, that lead to different influences on time and length scales is needed to further understand the fundamental factors driving these coupled pressure-fuel composition effects.

Introduction

This paper describes investigations of coupled fuel and pressure effects on the turbulent flame speed, $S_T$, of high hydrogen content (HHC) fuel blends, with particular emphasis on H$_2$/CO fuel blends [1, 2]. This work has been primarily motivated by the desire to gain deeper insight into the role played by preferential diffusion effects on turbulent flame propagation, a subject which has been well-documented in the literature [3-8]. For example, Wu et al. [3] demonstrated that two H$_2$/air mixtures, at $\phi = 0.80$ and 3.57, with the same un-stretched laminar flame speed, $S_{L,0}$, and operated with the same turbulence intensity, $u'_\text{rms}$, can have substantially different turbulent flame speeds. This showed that classical correlations of the form $S_T = f\left(u'_\text{rms},S_{L,0}\right)$ [9] did not adequately capture the sensitivities of the turbulent flame speed. Our own work has corroborated these findings for a broad range of H$_2$/CO mixtures for $u'_\text{rms}/S_{L,0}$ and pressures up to 45 and 5 atm respectively [1, 2].

Also of interest is the influence of coupled pressure and fuel composition effects on the turbulent flame speed, a subject for which there is no clear consensus in the literature. For example, turbulent consumption speed measurements of lean CH$_4$/air flames for a pressure range of 1-30 atm were reported by Kobayashi et al. [10]. It was concluded that $S_{T,GC}/S_{L,0}$ increased with pressure due to $S_{L,0}$ decreasing but that $S_{T,GC}$ itself was independent of pressure. Kitagawa et al. [11] reported
measurements of turbulent flame speeds of H₂/air mixtures at pressures ranging from 1-5 atm. They found that pressure had an influence on \( S_f / S_{L,0} \) through the pressure effect on \( S_{L,0} \). However, the influence on \( S_f \) is unclear. Daniele et al. [12] reported that \( S_{T,GC} / S_{L,0} \) increased with pressure at each given H₂/CO ratio and \( \nu_{\text{ms}} / S_{L,0} \) value. They also found that \( S_{T,GC} \) increased slightly with pressure for a fixed H₂/CO ratio and equivalence ratio.

Returning to the fuel effects, the observed sensitivities to the fuel composition have been attributed to the non-unity Lewis number and preferential diffusion effects which arise due to the coupling between heat and reactant diffusion and the multi-dimensional and unsteady turbulent flame and flow field [13]. These effects are collectively referred to as "stretch effects", and result in significant variations in the burning rate along the flame front [13]. For example, the H₂/CO mixtures investigated in our prior studies are highly stretch sensitive because of the high mass diffusivity of H₂. Stretch effects have been incorporated into models for the turbulent flame speed through a “Lewis number” [14, 15] or a mixture Markstein length [16, 17]. However, these approaches have some limitations. Attempting to capture just Lewis number effects neglects preferential diffusion effects which may have significance in multi-component fuels such as the H₂/CO mixtures considered in this work. On the other hand, the Markstein length, indicated in Figure 1(b) and calculated using Eq. (1), describes the low stretch rate sensitivity of the mixture, and accounts for both Lewis number and preferential diffusion effects.

\[
I_M = \left. \frac{\partial S_L}{\partial \nu} \right|_{\nu=0}
\]

Data sets taken at a given pressure, e.g., Wu et al. [3] or Venkateswaran et al. [1], demonstrate that, for negative Markstein length fuel/air mixtures, the turbulent flame speed increases as the absolute value of the Markstein length increases. However, a Markstein length scaling does not seem consistent with available data and data presented in this paper, since increasing pressure decreases Markstein lengths by thinning the flame, as seen in Figure 1(b). As described above, different measurements have shown that pressure both increases and has no effect on turbulent burning velocities [10, 12]; a Markstein length scaling argument would predict that increasing pressure, and thereby decreasing Markstein lengths, as shown in the calculations in Figure 1, should decrease turbulent burning velocities.

A promising approach to capture coupled fuel and pressure effects is based on leading points concepts [4, 18, 19]. Leading points are defined as the positively curved points on the turbulent flame front that propagate out furthest into the reactants. For negative Markstein length fuel blends, the local burning velocity is enhanced through stretch effects at these points [13], and it has been hypothesized that the dynamics of these leading points control the overall propagation rate [4, 19].

In our earlier work we developed a simple scaling law using quasi-steady leading points concepts [1]. Starting from a simple model problem of a flat flame propagating into a spatially varying iso-density mean flow field, we showed that after an initial transient, the flame propagated at a steady-state velocity equal to the burning velocity at the leading points. Furthermore, using the level-set equation, it was demonstrated for this model problem that the stable, steady-state burning velocity of the leading point is equal to the maximum stretched flame speed, \( S_{L,\text{max}} \), which is indicated in Figure 1(a) [1].
Figure 1: Stretch sensitivity calculations for (a) different H₂/CO mixtures of constant S_{L,0} at 1 atm and (b) a 50/50 H₂/CO mixture at constant S_{L,0} across different pressures.

Phenomenologically extending these model problems to a turbulent, reacting flow led to the following expression for the turbulent flame speed:

\[
\frac{S_T}{S_{L,\text{max}}} \leq 1 + \frac{u'_{\text{rms}}}{S_{L,\text{max}}}
\]  

The inequality in Eq. (2) can be replaced by an equality when the turbulent fluctuations evolve slowly relative to the time taken for the leading point burning velocity to reach \(S_{L,\text{max}}\).

Normalizing the flame speed data acquired by the authors by \(S_{L,\text{max}}\) showed a good collapse of the data in certain situations [1, 2]. For instance, data at constant pressure was shown to collapse very well across the range of the turbulence intensities and H₂/CO ratios. However, the scaling law was unable to collapse the data across pressures, which was attributed to non-quasi-steady, length scale, or some other Reynolds number effect. In our prior study, we particularly focused on the non-quasi-steady aspects of the problem, since \(S_{L,\text{max}}\) is itself a frequency dependent quantity [20] and estimates for the flow time-scale showed values that were easily on the order of the chemical time-scale[2]. As a result, if the unsteady stretching of the flame leading points happens faster than the flame can respond, this leads to the local burning velocity at the leading point to be lower than its quasi-steady \(S_{L,\text{max}}\) value. These effects can be incorporated into Eq. (2) by introducing \(s_{L,\text{max}}(\omega)\), which is the frequency dependent \(S_{L,\text{max}}\), to obtain:

\[
\frac{S_T}{S_{L,\text{max}}} = \frac{S_{L,\text{max}}(\omega)}{S_{L,\text{max}}} + \frac{\left<u'_{\text{rms}}\right>_{L_P}}{S_{L,\text{max}}}
\]  

The degree of non-quasi-steadiness can be parameterized using a Damköhler number defined as 

\[Da = \tau_{S_{L,\text{max}}} / \tau_{\text{flow}}.\]  

We would then expect that \(s_{L,\text{max}}(\omega)/S_{L,\text{max}} \rightarrow 1\) as \(\tau_{S_{L,\text{max}}} / \tau_{\text{flow}} \rightarrow 0\). In our previous works, we showed that for constant \(u'_{\text{rms}}/S_{L,\text{max}}\), the normalized turbulent flame speed correlated well with the Damköhler number, as expected from Eq. (3), suggesting that a two-parameter scaling given by Eq. (4) may be a promising approach to model the turbulent flame speed [2].
\[ \frac{S_c}{S_{L,\text{max}}} = f \left( \frac{u'_{\text{rms}}}{S_{L,\text{max}}}, \tau_{s_{L,\text{rms}}} \right) \]  

(4)

In order to more fully explore the adequacy or shortcomings of these scalings, this paper presents results of applying these leading points analyses to relevant data sets from the literature, including new data at pressures up to 20 atm obtained in our group.

**Parameter Calculation Approaches**

This section describes the approaches used to calculate the chemical kinetic parameters needed as inputs to the correlations. Estimates of \( S_{L,0} \) and \( S_{L,\text{max}} \) were determined using the PREMIX [21] and the extinction simulator in the OPPDIF [22] module in CHEMKIN respectively. For pure \( \text{H}_2 \) and \( \text{H}_2/\text{CO} \) mixtures the Davis mechanism [23] was utilized, while for \( \text{CH}_4 \) containing mixtures the GRI 3.0 mechanism [24] was utilized.

It is important to recall that the laminar flame speed can be defined as either a displacement or consumption speed [25]. In the un-stretched case, both definitions yield identical values, but for highly stretched flames the values can differ appreciably. In the following sections, both definitions are utilized to define \( S_{L,\text{max}} \). The displacement definition, \( S_{L,D,\text{max}} \), is obtained by defining the laminar flame speed as the minimum axial velocity just upstream of the reaction zone [26]. The consumption definition, \( S_{L,HR,\text{max}} \), is obtained from defining the laminar flame speed using Eq. (5), where \( q \) and \( h_{\text{sens}} \) are the heat release rate and sensible enthalpy respectively.

\[ S_{L,HR} = \int_{-\infty}^{\infty} \frac{\dot{q} dx}{\rho_a \left( h_{\text{sens},\infty} - h_{\text{sens},-\infty} \right)} \]  

(5)

A chemical time-scale associated with these highly stretched flamelets, \( \tau_{s_{L,\text{rms}}} \), is estimated by \( \tau_{s_{L,\text{rms}}} = \delta_f \left| \frac{S_{L,\text{rms}}}{S_{L,\text{max}}} \right| \) where \( \delta_f \big|_{s_{L,\text{rms}}} \) is the flame thickness at \( S_{L,\text{max}} \) calculated using

\[ \delta_f = (T_a - T_e) \left( \frac{dT}{dx} \right)_{\text{max}}. \]

It is important to recognize that the conditions for which turbulent flame speed data are available may be outside the conditions for which the mechanism has been optimized and so the uncertainty associated with the computed values of laminar flame properties is large. For instance, the Davis mechanism [23], used in calculations for \( \text{H}_2/\text{CO} \) mixtures in this paper, has been optimized by using flame speed data targets obtained at primarily 1 atm. A few \( S_{L,0} \) targets at 15 atm were also used for primarily rich \( \text{H}_2/\text{air} \) mixtures. The lack of flame speed targets at higher pressures for lean hydrogen containing mixtures is due to the fact that these flames typically exhibit cellular structures rendering the determination of \( S_{L,0} \) impossible [13]. This point should be taken in mind when examining some of the high pressure data presented in the next section, which show values of \( u'_{\text{rms}}/S_{L,0} > 10^3 \).

Finally, in our calculations of the Damköhler number, we have used the time-scale associated with the Taylor scale, \( l_\lambda \), as one characteristic fluid mechanical time-scale. The scaling for the Taylor length scale is given in Eq. (6) [27].

\[ l_\lambda = l_{\text{int}} \sqrt{10 \, \text{Re}}^{-1/2} \]  

(6)
The Taylor time-scale, $\tau_d$, is then calculated as $l_d/u'_{max}$. In Eq. (6), $l_{int}$ is the integral length scale, which is derived from a measured integral time-scale by invoking Taylor’s hypothesis [27]. In the cases where integral scales were not available, $l_{int}$ was estimated using a geometric scale such as the burner diameter.

**Results and Analysis**

In this section we report measurements and leading points correlations of turbulent consumption speed measured by our group and turbulent flame speed data reported in the literature. Table 1 below is a summary of the data sets considered in this paper along with the legend employed for the plots. Recognizing that the turbulent flame speed is a definition and system dependent parameter [28], comparisons are only made within groupings of data measured in similar configurations where the turbulent flame speeds are defined in a similar manner. In all, we utilized data corresponding to roughly 1000 measurement points, obtained at roughly 120 different pressures and mixture compositions, for which we performed detailed kinetic calculations to determine the corresponding $S_{L,0}$, $S_{L,max}$ and $\tau_{S_{L,max}}$ values.
Table 1: Summary of turbulent flame speed data sets analyzed in this paper along with plotting legends used in this paper.

<table>
<thead>
<tr>
<th>Database</th>
<th>Configuration</th>
<th>Turbulent Flame Speed Definition</th>
<th>Experiment</th>
<th>Plot Legend and Experimental Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Georgia Tech database including Ref.[1, 2]</td>
<td>Bunsen</td>
<td>Global consumption, $S_{T,GC}$</td>
<td>Constant $S_{L,0}$</td>
<td><img src="image" alt="Constant $S_{L,0}$ Legend" /></td>
</tr>
<tr>
<td>PSI database, Ref.[12]</td>
<td>Bunsen</td>
<td>Global consumption, $S_{T,GC}$</td>
<td>Varying $S_{L,0}$</td>
<td><img src="image" alt="Varying $S_{L,0}$ Legend" /></td>
</tr>
<tr>
<td>Kido et al.[29]</td>
<td>Spherical bomb</td>
<td>Global consumption, $S_{T,GC}$</td>
<td>Constant $S_{L,0}$</td>
<td><img src="image" alt="Constant $S_{L,0}$ Legend" /></td>
</tr>
<tr>
<td>Nakahara et al.[30]</td>
<td>Spherical bomb</td>
<td>Global consumption, $S_{T,GC}$</td>
<td>Constant $S_{L,0}$</td>
<td><img src="image" alt="Constant $S_{L,0}$ Legend" /></td>
</tr>
<tr>
<td>Kitagawa et al.[11]</td>
<td>Spherical bomb</td>
<td>Global consumption, $S_{T,GC}$</td>
<td>Varying $S_{L,0}$</td>
<td><img src="image" alt="Varying $S_{L,0}$ Legend" /></td>
</tr>
<tr>
<td>LSB database, Ref.[31-33]</td>
<td>Low swirl burner</td>
<td>Local displacement, $S_{T,LD}$</td>
<td>Varying $S_{L,0}$</td>
<td><img src="image" alt="Varying $S_{L,0}$ Legend" /></td>
</tr>
</tbody>
</table>
**Bunsen Data**

*Georgia Tech database*

Since the experimental facility and data reduction techniques are described in extensive detail in our earlier works [1, 2], only a brief overview is presented here. The burner configuration utilized is a piloted Bunsen burner, which is housed inside a fully remotely operable and optically accessible pressure vessel. The burner is contoured to inhibit boundary layer growth and to ensure top hat profiles in the mean and fluctuating velocities at the exit. The flat stoichiometric CH₄/air pilot flame is stabilized on a sintered plate fixed around the burner nozzle.

Measurements of $S_{T,GC}$ have been obtained at pressures of 1-20 atm as a function of $u_{in}/S_{L,0}$ using the 12 and 20 mm diameter Bunsen burners for a reactant temperature of 300 K. Data were acquired at mean flow velocities of 4-50 m/s and H₂/CO ratios ranging from 30/70-90/10 by volume.

Two sets of experiments were conducted. In the first set, data were obtained where the mixture $S_{L,0}$ was kept nominally constant at 34 cm/s by adjusting the equivalence ratio at each H₂/CO ratio. These experiments were conducted at pressures up to 10 atm with the 12 mm diameter burner. In the second of experiments, which were conducted with the 20 mm diameter burner, the H₂/CO ratio, equivalence ratio and pressure were all varied such that $S_{L,0}$ did not remain constant. Table 1 summarizes the experimental conditions and plotting legends employed for these data and Figure 2 summarizes where all the measured data are located on a Borghi-Peters diagram [34, 35].

![Figure 2: Location of all the data reported in this study (12 and 20 mm) on the Borghi-Peters diagram.](image)

Figure 3 plots $S_{T,GC}$ as a function of $u_{in}$, normalized by $S_{L,0}$ and $S_{L,HR,max}$ for the constant $S_{L,0}$ studies performed using the 12 mm diameter burner for the mixtures and conditions reported in Table 1.
Figure 3: $S_{T,GC}$ as a function of $u'_{rms}$ normalized by (a) $S_{L,0}$ and (b) $S_{L,HR,max}$ for the constant $S_{L,0}$ studies conducted using the 12 mm diameter burner (See Table 1 for legend)

Focusing first on the $S_{L,0}$ normalized data, we can few interesting observations from Figure 3(a). First, note that the ‘fuel effects’ discussed in the introduction are present at all pressures. In other words, different H$_2$/CO blends at constant $S_{L,0}$ and $u'_{rms}$ have different turbulent flame speeds. In our earlier work we made similar observations but for data up to 5 atm [2]. Second, increasing pressure leads to increased turbulent flame speed i.e. at constant $S_{L,0}$ and $u'_{rms}$, $S_{T,GC}$ increases with pressure. This increase is quantified in Figure 4, which plots the ratio of $S_{T,GC}$ at 5 and 10 atm to 1 atm for each mixture and mean flow velocity as a function of turbulence intensity. This ratio has values of about 1.8-2.1 and 2.2-2.5 at 5 and 10 atm, respectively. Note that this is not an $S_{L,0}$ effect, as $S_{L,0}$ is kept fixed at 34 cm/s.

Figure 3(b) then plots the result of scaling this data using Eq. (2). We note that the 10 atm, 5 atm and 1 atm data sets collapse quite well individually when normalized by $S_{L,HR,max}$, but that there are systematic differences between them.

Figure 4: Ratio of $S_{T,GC}$ at 5 and 10 atm to 1 atm across the range of turbulence intensities investigated.

In Figure 5 we plot, on linear and log-log scales, $S_{T,GC}$ data acquired using the 20 mm burner at pressures up to 20 atm for the second set of experiments described earlier. The mixtures and conditions are summarized in Table 1.
Figure 5: $S_{T,GC}$ as function of $u'_{rms}$, normalized by $S_{L,0}$ at various mean flow velocities, H$_2$/CO ratios, and pressures for the 20 mm diameter burner plotted on a (a) linear and (b) log-log scale. (See Table 1 for the legend).

Note that data have been acquired at $u'_{rms}/S_{L,0}$ values of up to almost 1800. However, as discussed earlier, these large values in the normalized turbulence intensity and turbulent flame speed are likely highly inaccurate due to the large uncertainties in the calculated values of $S_{L,0}$.

Figure 6(a) plots the result of normalizing all the 20 mm diameter burner data acquired at 1 atm by $S_{L,H R,max}$. This data had the interesting behavior that all the data collapsed except for the 30 m/s CH$_4$/air case [1]. Figure 6(a) has also been reproduced here because it is difficult to see this collapse in Figure 6(b), which plots the entire $S_{L,H R,max}$ normalized 20 mm diameter burner data set.

From Figure 6(b), we can make similar observations regarding the data of a given pressure collapsing reasonably well, but not collapsing across pressures. However, this trend is not as clear-cut as in Figure 3(b), because of the broad range in $S_{L,0}$ and $S_{L,max}$ present in this data set.

Following the time-scale discussion in the introduction, Figure 7 plots $S_{T,GC}/S_{L,H R,max}$ as a function of $\tau_{S_{L,H R,max}}/\tau_{flow}$, at two representative fixed turbulence intensity conditions, $u'_{rms}/S_{L,H R,max}$ of
7 and 13.5 from Figure 3(b). In Figure 7(a), \( \tau_{\text{flow}} \) is defined as a bulk flow time-scale \( \tau_B = D/U_0 \) and as a Taylor time-scale \( \tau_\lambda = l_\lambda/u'_{\text{rms}} \) in Figure 7(b).

![Figure 7: Dependence of \( \frac{S_{\text{TGC}}}{S_{L,HR,\text{max}}} \) upon \( \frac{\tau_{SL,HR,\text{max}}}{\tau_B} \) at fixed turbulence intensities, \( \frac{u'_{\text{rms}}}{S_{L,HR,\text{max}}} = 3.5 \) and 6.5 for the 12mm diameter burner where \( \tau_{\text{flow}} \) is scaled as (a) \( D/U_0 = \tau_B \) and (b) \( l_\lambda/u'_{\text{rms}} = \tau_\lambda \).

Note from Figure 7 the clear expected correlation between the normalized turbulent flame speed and time-scale ratio across the entire range of pressure and fuel compositions. Also, note the difference in the normalized time-scale ranges between Figure 7(a) and Figure 7(b), which utilize a bulk fluid mechanical time-scale and Taylor flow time-scale respectively. In Figure 7(b) the range of normalized chemical time-scales has decreased compared to in Figure 7(a), which can be attributed to the fact that although the chemical time-scale decreases with pressure as \( \tau_{s,\text{max}} \sim p^{-1} \) [2], the Taylor time-scale decreases with pressure as \( \tau_\lambda \sim p^{-3/2} \). Also note that \( \frac{\tau_{s,\text{max}}}{\tau_{\text{flow}}} \) is less than 1 in Figure 7(a), but greater than 1 in Figure 7(b) suggesting that the leading point is quasi-steady with respect to largest turbulent scales, but non-quasi-steady with respect to the Taylor time-scale.

Similar time-scale analyses were performed for the 20 mm data and the results are shown in Figure 8. The same trends seen in Figure 7(a) and Figure 7(b) are also seen here, namely, the clear correlation between the turbulent flame speed and the time-scale ratio across the range of mixtures and conditions.
Figure 8: Dependence of $S_{T,GC}/S_{L,HR,max}$ upon $\tau_{S_{L,HR,max}}/\tau_B$ at fixed turbulence intensities, $u'_{rms}/S_{L,HR,max} = 5$ and 14 for the 20 mm diameter burner where $\tau_{flow}$ is scaled as (a) $D/U_0$, and as (b) $l_j/u'_{rms}$.

**PSI database**

This section presents the turbulent consumption speed database acquired by Daniele et al. [12] at the Paul Scherrer Institute (PSI). The experimental configuration consists of a straight tube of 25 mm diameter that exits into a dump geometry. The database of $S_{T,GC}$ measurements were acquired for H$_2$/CO mixtures of 33/67, 50/50 and 67/33 at a reactant temperature of 623 K and pressures ranging from 1–20 atm at a mean flow velocity of 40 m/s. Table 1 summarizes the mixtures explored in this work, along with the legend used to plot the data.

Figure 9(a) presents a slightly modified version of the data reported in Ref. [12]. In Ref. [12], the turbulence intensity used to correlate the $S_{T,GC}$ data was determined at the intersection of the flame centerline and the $\langle c \rangle = 0.05$ progress variable. The Georgia Tech data is plotted using the turbulence intensity measured at the center of the burner exit. To maintain consistency, the PSI data is re-plotted here using the turbulence intensity at the center of the burner exit. In their work, since the turbulence generating device and the mean flow velocity of 40 m/s were unchanged across all conditions, the turbulence intensity, $u'_{rms}$, at the burner exit is assumed to be the same, at 2.8 m/s, across all conditions. As a result, the observed variation in $u'_{rms}/S_{L,0}$ is due to the changing $S_{L,0}$.

Figure 9(b), which plots the results of normalizing the data by $S_{L,HR,max}$, shows that the scaling given by Eq. (2) does not appear to collapse the data as well compared to the Georgia Tech 12 mm or the 1 atm 20 mm data. However, it is not completely surprising that this data does not appear to collapse very well, since similar to the Georgia Tech 20 mm data, this data spans a wide range of $S_{L,0}$ and $S_{L,max}$. 

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Employing the time-scale analysis for fixed turbulence intensities, \( u'_{\text{rms}} / S_{L,0} \), of 2.5 and 6.5, we note similar correlation between the turbulent flame speed and critically stretched chemical time-scale ratio in Figure 10 that we noted with our data. In these calculations, \( \tau_{\text{flow}} \) is scaled as \( D/U_0 \). Although the results are not shown here, we repeated the analysis using the Taylor time-scale as the characteristic fluid mechanical time-scale. We observed that the normalized time–scale were on the order of one, suggesting that the leading points may not be quasi-steady steady with respect to the Taylor scale.

**Spherical bomb database**

The database of fan-stirred spherical bomb experiments that are analyzed in this work is comprised of data from Ref. [11, 29, 30]. Although there are many other spherical bomb studies reported in the literature [16, 17, 36], these data sets have been selected since they investigate fuel and coupled fuel and pressure effects. In Ref. [29], \( S_{T,GC} \) data was obtained for H\(_2\)/O\(_2\)/N\(_2\) and
CH4/O2/N2 mixtures for equivalence ratios, \( \phi = 0.7, 0.9 \) and 0.98. The O2/N2 and equivalence ratios were selected so as to keep the mixture \( S_{L,0} \) constant at 15 cm/s. In Ref. [30], \( S_{T,GC} \) for various H2/CH4 mixtures whose \( S_{L,0} \) was held constant at 15 and 25 cm/s was obtained. The \( S_{L,0} \) was held fixed by varying the equivalence ratio and O2/N2 ratio. Finally, in Ref. [11], turbulent flame speed data were obtained for H2/air mixtures for equivalence ratios and pressures ranging from 0.4-1.0 and 1–5 atm respectively. The reader is referred to the cited papers for more details regarding the experimental facility and data acquisition and reduction.

Figure 11 plots the spherical bomb database in both \( S_{L,0} \) and \( S_{L,D,max} \) normalized forms. From Figure 11(b), we observe that the \( S_{L,D,max} \) normalization collapses the data reasonably well, particularly at low \( u'_{rms}/S_{L,D,max} \). However, the scatter in the normalized data increases with increasing \( u'_{rms}/S_{L,D,max} \).

![Figure 11: \( S_{T,GC} \) as a function of \( u'_{rms} \) normalized by \( S_{L,0} \) and \( S_{L,D,max} \) for the spherical bomb database. Refer to Table 1 for the plotting legend.](image)

In Figure 12, we investigate the source of scatter in the high \( u'_{rms}/S_{L,D,max} \) region using the time-scale analysis. The analysis is conducted at turbulence intensities, \( u'_{rms}/S_{L,D,max} \) of 2.5 and 5.0 and for this analysis, \( \tau_{flow} \) is defined as \( \tau_{int}/u'_{rms} \). These \( u'_{rms}/S_{L,D,max} \) values only contain data points obtained at atmospheric pressure. From Figure 12, we note the familiar decreasing trend of \( S_{T}/S_{L,D,max} \) with the normalized leading point chemical time-scale consistent with the expectations derived from Eq.(3). In addition, the normalized leading point chemical is uniformly less than 1 suggesting that the leading point is quasi-steady with respect to the large turbulent scales. However, although not shown here, we observe that the leading points are non-quasi-steady with respect to the Taylor time-scale.
LSB database

This section presents correlations of local turbulent displacement speed, $S_{T,LD}$, measurements acquired using the low-swirl burner (LSB) [31-33]. These measurements were obtained at 1 atm and 300 K for the variety of mixtures outlined in Table 1. $S_{T,LD}$ is defined as the local mean axial velocity at the location where the centerline mean axial velocity profile deviates from a linear profile [37]. Figure 13 plots $S_{T,LD}$ as a function of $u'_{rms}$ normalized by both $S_{L,0}$ and $S_{L,D,max}$. From Figure 13(b) shows that there is still significant scatter present when the data is scaled using Eq.(2).

Performing the time-scale analysis with the LSB data set, we see in Figure 14 the previously observed decreasing trend in $S_{T,LD}/S_{L,max}$ with the normalized leading point chemical time-scale. In this analysis $\tau_{flow}$ is defined as $R_i/U_0$ where $R_i$ is the outer radius of the LSB, which is taken to be 3.17 cm. Although not shown here, when we scale $\tau_{flow}$ using the Taylor time-scale, we see that the
normalized time-scales are still less than one, which suggests that for these data the leading point is quasi-steady with respect to the Taylor scales.

Figure 14: Dependence of $S_{L,D,\text{max}}$ upon $\tau_{L,D,\text{max}}/\tau_{\text{flow}}$ at fixed turbulence intensities, $u'_{\text{rms}}/S_{L,\text{max}}$ of 1.1 and 1.6 for the low swirl burner data set.

**Conclusions**

In this paper, we show that a broad range of fuel composition and pressure data can be correlated with the maximum laminar flame speed, $S_{L,\text{max}}$, and a chemical time-scale ratio scaling. In particular, it was suggested that pressure effects influence the turbulent burning velocity by altering how well the flame's internal chemistry can track the time varying stretch rate at the leading point. If this assertion is true, then it clearly indicates the strong coupling effects of stretch and pressure. In particular, a key feature of this argument is that, in flames where $S_{L,\text{max}}/S_{L,0} >> 1$, then non-quasi-steady effects can significantly alter the burning velocity of the leading point. This argument also suggests then, that non-quasi-steady effects should have much less effect on mixtures with weak stretch sensitivity, where $S_{L,\text{max}}/S_{L,0} \sim 1$.

Returning to the Georgia Tech data, it must also be emphasized that the data, when normalized by $S_{L,\text{max}}$, showed a clear correlation with pressure. Thus, any parameter that also correlates with pressure will also do a reasonable job of scaling the pressure effects – and, there are other significant parameters that do so. In particular, the Reynolds number linearly increases with pressure. As a result, these pressure effects could also be correlated with Reynolds number. For similar reasons, the pressure effects can also be scaled with certain length scale ratios, as both the Taylor and Kolmogorov length scales also have a Reynolds number. Thus, these pressure effects could also be scaled using a length scale ratio. To illustrate, Figure 15 plots $S_{T,GC}/S_{L,HR,\text{max}}$ as a function of the normalized Taylor length scale at two constant $u'_{\text{rms}}/S_{L,HR,\text{max}}$ values from the Georgia Tech 12 mm data set.

In Figure 15 we see that the $S_{T,GC}/S_{L,HR,\text{max}}$ decreases with the normalized Taylor length scale, which can be explained using Reynolds number arguments. As per Eq. (6), for a fixed large scale, smaller Taylor scales are due to larger turbulent Reynolds numbers which increases $S_{T,GC}/S_{L,HR,\text{max}}$ [38, 39].
Figure 15: $S_{T.GC}/S_{L.HR,\text{max}}$ as a function of the normalized Taylor length scale at constant $u'_{\text{rms}}/S_{L.HR,\text{max}}$ values of 7 and 13.5 for the 12 mm diameter dataset for the Georgia Tech database.

However, it must be noted that the LSB and spherical bomb databases exhibited clear correlations with the normalized leading point time-scale despite the fact these data points were all obtained at atmospheric pressures. This suggests that two parameter scaling given by Eq. (4) captures the leading point dynamics as well as non-quasi-steady-state effects. However, additional data is needed to differentiate between chemical time and Reynolds number effects and future work will focus obtaining data over a broadened set of experimental conditions that will enable differentiation between time-scale, length scale, and Reynolds number effects on the turbulent burning velocity.

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**References**


